## Summit Public Schools Summit, New Jersey

**Grade Level / Content Area:** 10-11 / Mathematics

Length of Course: 1 Year

Algebra 2/Trigonometry

## **Course Description:**

The overall goals of the course are: to explore a variety of functions that can be used to model relationships between sets of numbers; to introduce the set of complex numbers; to build equation-solving skills; to introduce basic data analysis and probability. Students will be expected to work with relations that are in a variety of representations, including algebraic, tabular, graphic, and verbal forms. Real-world data will be used to motivate and extend all topics. The TI-83 graphing calculator and web-based technologies will be used extensively to assist in solving complicated problems. Students will be expected to communicate mathematics clearly in written, verbal, and algebraic forms.

## **Course Pacing:**

1.	Review of Basic Algebra / Linear and Absolute Value Equations and	
	Inequalities (Chapters 1 and 2)	13 days
2.	Systems of Linear Equations and Inequalities (Chapter 3)	13 days
3.	Quadratic Equations and Parabolas (Chapter 5)	22 days
4.	Functions (Chapter 6)	10 days
5.	Powers, Roots, and Radicals (Chapter 7)	12 days
6.	Exponential and Logarithmic Functions (Chapter 8)	16 days
7.	Polynomials (Chapter 9)	15 days
8.	Trigonometric Ratios/Functions and Graphs (Chapter 13/14)	16 days
9.	Trigonometric Oblique Triangles, Inverses, Identities, and	
	Equations (Chapters 13/14)	16 days

Revised (2019) by:

Cheryl Adair

**Unit 1: Analyzing Equations and Inequalities** 

Topic	Section In Text	Time Frame	SWBAT
Sets of Real Numbers	1.1 1.2	1	<ul><li>Represent and classify real numbers</li><li>Evaluate algebraic expressions</li></ul>
Solving Linear and Literal Equations	1.3 1.5	1	<ul> <li>Solve a linear equation for a numerical value of "x"</li> <li>Manipulate a literal equation and solve for a specified variable</li> </ul>
Solving Linear/Compound inequalities and Interval Notation	1.6	1	<ul> <li>Solve a linear inequality and graph its solution on a number line</li> <li>Solve a compound inequality and graph its solution on a number line</li> <li>Use interval notation to represent solutions of linear inequalities in one variable</li> </ul>
Solving Absolute Value Equations and Inequalities	1.7	2	<ul> <li>Solve an absolute value equations by first transforming it into two linear equations</li> <li>Graph the solution set to an absolute value inequality on a number line</li> <li>Solve an absolute value inequality by first transforming it into a compound inequality</li> <li>Graph the solution set to an absolute value inequality on a number line</li> </ul>
Relations, Functions, Domain and Range	6.1	1	<ul> <li>Define function</li> <li>Recognize a function given a table, mapping diagram, graph (apply vertical line test), and a set of ordered pairs</li> <li>Use function notation</li> </ul>
Domain and Range of Graphs		1	Identify domain and range of discrete and continuous graphs and define using interval notation
Graphing Absolute Value with Transformations	2.6	2	<ul> <li>Graph the parent function of an absolute value function</li> <li>Graph absolute value functions with rigid transformations (up/down, left/right, upside down).         Understand how f(x+k), f(x) + k and -f(x) transform the graphs.     </li> <li>Find the domain and range in interval notation of these graphs</li> <li>Write an absolute value function given a graph</li> </ul>
Review/Test	Total	2 11	

+2 (extra
time/quizzes
= 13  days

#### UNIT 1 STANDARDS ADDRESSED

A.CED.1 Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.* 

A.CED.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

A.CED.3 Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.

A.CED.4 Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. For example, rearrange Ohm's law V = IR to highlight resistance R.

#### CCSS.MATH.CONTENT.HSF.IF.C.7.B

Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.

#### CCSS.MATH.CONTENT.HSF.BF.B.3

Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.

## **Unit 2: Systems of Equations and Inequalities**

Topic	Section	Time Frame	SWBAT
	In Text		

Solving Systems by	3.1	1	Solve a system of linear equations graphically
Graphically and Algebraically	3.2		<ul> <li>Recognize when a system has no solution or infinitely many based on its graph (parallel vs. same line)</li> </ul>
ger a en y			Solve systems of linear equations by finding intersections on graphing calculator
			<ul> <li>Be able to solve word problems involving systems of linear equations by using substitution and elimination</li> </ul>
			• Recognize when a system will have no solution or infinitely many solutions based on answer patterns (0=9, no solution, 9=9 infinitely many)
Word problems involving linear systems	3.3	2	Set up and solve a word problem involving systems of linear equations
Graphing Systems of Inequalities	3.4	1	
Linear Programming Method	3.5	1	Minimize and maximize an objective quantity
Problems solving using Linear Programming	3.5	2	Use linear programming to answer questions about real-life situations
Solving 3X3 Systems of	3.6	2	Solve a system of linear equations in three variables algebraically
Equations Algebraically and using RREF			<ul> <li>Solve a system of linear equations in three variables using matrices and RREF on the graphing calculator</li> </ul>
-			• Set up and solve a word problem involving systems of linear equations. Use the calculator to solve such problems.
Review/Test		2	
	Total	11	
		+2 extra	
		time/quizzes =	
		13 days	

## UNIT 2 STANDARDS ADDRESSED

CCSS.MATH.CONTENT.HSA.REI.D.10
Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).

## CCSS.MATH.CONTENT.HSA.REI.D.11

Explain why the x-coordinates of the points where the graphs of the equations y = f(x) and y = g(x) intersect are the solutions of the equation f(x) = g(x); find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where f(x) and/or g(x) are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.\*

## CCSS.MATH.CONTENT.HSA.REI.D.12

Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.

**Unit 3: Quadratic Equations and Parabolas** 

Topic	Section	Time Frame	SWBAT
	In Text		
Algebra 1 Review of	9.3	2	Factor GCF
factoring polynomials			<ul> <li>Factor difference of squares</li> </ul>
completely			• Factor trinomials
			Factor grouping
Factoring polynomials	9.3	.5	<ul> <li>Factor sum and difference of cubes</li> </ul>
using the sum and			
difference of cubes			
Solving Quadratic	5.1	1	<ul> <li>Use the zero product property as a method of solving quadratics</li> </ul>
Equations by Factoring			<ul> <li>Use taking square roots as a method of solving quadratics</li> </ul>
and Taking Square Roots			
Solving Quadratic	5.3	2	<ul> <li>Use completing the square as a method of solving quadratics</li> </ul>
Equations by			
Completing the Square			
Using the Discriminant	5.4	1	<ul> <li>Use the quadratic formula as a method of solving quadratics</li> </ul>
to Describe the Roots of			• Use the discriminate to find the number of solutions of a quadratic equation and to describe the
a Quadratic and Solving			roots
Quadratic Equations by			
the quadratic formula			

Choosing the Best Method to solve Quadratic Equations		1	
Graphing Parabolas in Intercept Form		.5	<ul> <li>Graph quadratic functions in intercept form</li> <li>Identify max and min values, x-intercepts, domain and range in interval notation.</li> </ul>
Graphing Parabolas in Standard Form	5.2	.5	<ul> <li>Graph quadratic functions in standard form</li> <li>Identify max and min values, x-intercepts, domain and range in interval notation.</li> </ul>
Graphing Parabolas in Vertex Form		.5	<ul> <li>Graph quadratic functions in vertex form</li> <li>Identify max and min values, x-intercepts, domain and range in interval notation.</li> </ul>
Use Completing the Square to Write the Equation of a Parabola in Vertex Form		1	Use completing the square to convert quadratics from standard to vertex form
Solving Non-linear Systems of Equations		2	<ul> <li>Solve systems of nonlinear (limit to abs, quadratic, simple rational) by finding intersections on calculator, or by recognizing that if there is no intersection point, the system has no solution.</li> </ul>
Solving Quadratic Word Problems	5.4 5.6	3	Use the algebraic methods learned to solve word problems (vertical motion model)
Complex Numbers	5.5 5.6	2	<ul> <li>Recognize when and in what context an imaginary numbers arise</li> <li>Apply arithmetic operations to complex numbers.</li> <li>Raise "i" to "high powers" (i^13)</li> </ul>
Review/Test		2	
	Total	19	
		+3 extra time/quizze s = 22 days	

#### UNIT 3 STANDARDS ADDRESSED

#### CCSS.MATH.CONTENT.HSA.SSE.A.1

Interpret expressions that represent a quantity in terms of its context.\*

#### CCSS.MATH.CONTENT.HSA.SSE.A.1.A

Interpret parts of an expression, such as terms, factors, and coefficients.

#### CCSS.MATH.CONTENT.HSA.SSE.A.2

Use the structure of an expression to identify ways to rewrite it. For example, see x4 - y4 as (x2)2 - (y2)2, thus recognizing it as a difference of squares that can be factored as (x2 - y2)(x2 + y2).

#### CCSS.MATH.CONTENT.HSF.IF.C.8.A

Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.

#### CCSS.MATH.CONTENT.HSF.IF.C.7.A

Graph linear and quadratic functions and show intercepts, maxima, and minima.

#### CCSS.MATH.CONTENT.HSF.IF.C.9

Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.

#### CCSS.MATH.CONTENT.HSF.BF.B.3

Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.

#### CCSS.MATH.CONTENT.HSN.CN.A.1

Know there is a complex number has such that i2 = -1, and every complex number has the form a + bi with a and b real.

## CCSS.MATH.CONTENT.HSN.CN.A.2

Use the relation i2 = -1 and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.

## CCSS.MATH.CONTENT.HSN.CN.C.7

Solve quadratic equations with real coefficients that have complex solutions.

## CCSS.MATH.CONTENT.HSN.CN.C.8

(+) Extend polynomial identities to the complex numbers. For example, rewrite  $x^2 + 4$  as (x + 2i)(x - 2i).

## CCSS.MATH.CONTENT.HSN.CN.C.9

(+) Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.

**Unit 4: Functions** 

Topic	Section	Time Frame	SWBAT
	In Text		
Cubic Function		1	Graph using transformation techniques
Transformations			
Operations with	6.2	2	• Perform operations with functions, state the domain of the resulting function (including union
Functions (with			of sets for rational functions, etc)
compositions)			<ul> <li>Perform a composition of two functions</li> </ul>
Even/Odd Functions		1	<ul> <li>Recognize whether a function is even, odd, or neither based on its graph</li> </ul>
			<ul> <li>Algebraically show that a function is even, odd, or neither</li> </ul>
Inverses	6.3	1	<ul> <li>Find the inverse of a function graphically and algebraically</li> </ul>
			Perform the horizontal line test
Piecewise/Step	6.4	2	Graph step and piecewise functions
Functions			<ul> <li>Use step and piecewise functions in real life situations</li> </ul>
Review/Test		2	
	Total	9	
		+1 extra	
		time/quizzes	
		$= 10^{\circ} days$	

## UNIT 4 STANDARDS ADDRESSED

## CCSS.MATH.CONTENT.HSF.IF.C.7

Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.\*

## CCSS.MATH.CONTENT.HSF.IF.C.7.B

Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.

#### CCSS.MATH.CONTENT.HSF.BF.B.3

Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.

#### CCSS.MATH.CONTENT.HSF.BF.B.4

Find inverse functions.

#### CCSS.MATH.CONTENT.HSF.BF.B.4.A

Solve an equation of the form f(x) = c for a simple function f that has an inverse and write an expression for the inverse. For example,  $f(x) = 2 \times 3$  or f(x) = (x+1)/(x-1) for  $x \ne 1$ .

#### CCSS.MATH.CONTENT.HSF.BF.B.4.B

(+) Verify by composition that one function is the inverse of another.

#### CCSS.MATH.CONTENT.HSA.REI.D.11

Explain why the x-coordinates of the points where the graphs of the equations y = f(x) and y = g(x) intersect are the solutions of the equation f(x) = g(x); find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where f(x) and/or g(x) are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.\*

#### CCSS.MATH.CONTENT.HSF.IF.B.4

For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.\*

## CCSS.MATH.CONTENT.HSF.IF.B.5

Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.\*

**Unit 5: Radicals/Rational Exponents** 

Topic	Section In Tout	Time Frame	SWBAT
Dunanting	In Text	1	C'
Properties of	7.1	1	<ul> <li>Simplify using all properties of exponents</li> </ul>
Exponents			
Fractional Roots	7.3	2	• Evaluate nth roots of real numbers using radical notation and rational exponent notation both by hand and with a calculator
Simplifying Radical	7.4	2	Multiply and divide radicals
Expressions			<ul> <li>Simplify radicals using absolute value where appropriate</li> </ul>
			<ul> <li>Define radicand, index, like radicals</li> </ul>
Multiplying Radical			Add and subtract radicals
Expressions			Rationalize denominators using conjugate
Dividing Radical Expressions			
Solving Radical	7.5	2	Solve radical equations
Equations			• Find extraneous solutions and reason about what it means to be an extraneous solution
Graph	7.6	1	Graph cubic functions with transformations
Cubic/Cube			<ul> <li>Find domain and range of cubic functions in interval notation</li> </ul>
Root/Square root			<ul> <li>Connect inverses</li> </ul>
Functions			<ul> <li>Justify whether certain cubic graphs are even, odd, or neither</li> </ul>
Review/Test		2	
	Total	10	
		+2	
		(for additional	
		time/quizzes)	
		12 days	

#### **UNIT 5 STANDARDS ADDRESSED**

## CCSS.MATH.CONTENT.HSA.REI.A.2

Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.

#### CCSS.MATH.CONTENT.HSN.RN.A.1

Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. For example, we define  $5^{1/3}$  to be the cube root of 5 because we want  $(5^{1/3})^3 = 5^{(1/3)3}$  to hold, so  $(5^{1/3})^3$  must equal 5.

#### CCSS.MATH.CONTENT.HSN.RN.A.2

Rewrite expressions involving radicals and rational exponents using the properties of exponents.

#### CCSS.MATH.CONTENT.HSN.RN.B.3

Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.

#### CCSS.MATH.CONTENT.HSF.IF.C.7.B

Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.

**Unit 6: Exponential and Logarithmic Functions** 

Topic	Section	Time	SWBAT
	In Text	Frame	
Graphs of Exponential Functions	8.1	1	<ul> <li>Graph exponential functions</li> <li>Find the domain and range in interval notation</li> </ul>
Exponential Equations not requiring logs		1	Solve exponential equations both with like and unlike bases that do not require logs
Logarithms	8.2	2	Switch between logarithmic and exponential form

			<ul> <li>Evaluate logarithms without the calculator</li> <li>Evaluate logarithms using the change of base formula and the calculator</li> <li>Graph logarithmic functions</li> </ul>
Properties of Logarithms	8.3	2	Use properties to expand and condense logarithms
Solving Equations Using Logarithms	8.6	2	Solve logarithmic equations and exponential equations requiring logarithms with all bases.
Solving Equations Using Natural Logarithms	8.4 8.6	1	• Evaluate, expand, and condense logarithms of base e
Evaluating and Graphing Natural Logarithms	8.5	1	<ul> <li>Graph natural logarithms</li> <li>Use logarithmic properties to evaluate natural logarithms</li> </ul>
Word Problems with Logarithms	8.5 8.6	2	• Solve word problems related to logarithms (compound interest, growth and decay word problems)
Review / Test		2	
	Total	14 +2 (for additional time/ quizzes) 16 days	

## UNIT 6 STANDARDS ADDRESSED

## CCSS.MATH.CONTENT.HSA.SSE.A.1.B

Interpret complicated expressions by viewing one or more of their parts as a single entity. For example, interpret P(1+r)n as the product of P and a factor not depending on P.

## CCSS.MATH.CONTENT.HSF.IF.C.7.E

Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.

#### CCSS.MATH.CONTENT.HSF.IF.C.8.B

Use the properties of exponents to interpret expressions for exponential functions. For example, identify percent rate of change in functions such as  $y = (1.02)^t$ ,  $y = (0.97)^t$ ,  $y = (1.01)12^t$ ,  $y = (1.2)^t/10$ , and classify them as representing exponential growth or decay.

#### CCSS.MATH.CONTENT.HSF.BF.B.5

(+) Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.

## CCSS.MATH.CONTENT.HSF.LE.A.4

For exponential models, express as a logarithm the solution to abct = d where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology.

## **Unit 7: Exploring Polynomial Functions**

Topic	Section	Time Frame	SWBAT
	In Text		
Monomials/Polynomials/	9.1	1	Recognize characteristics of a polynomial (degree, leading coefficient, standard form, monomial,
Classifying Polynomials			binomial, trinomial, etc)
			Add, subtract, and multiply polynomials
Solving polynomial	9.3	1	Finding all roots of a polynomial equation of higher degree using factoring/quadratic formula
equations of higher degree			
Long and Synthetic	9.4	2	Divide polynomials using long and synthetic division
Division of Polynomials			Recognize when to use synthetic division over long division
Graphs of Polynomial	9.2	2	• Find # of max turns (degree-1)
Functions			Find relative extrema on calculator

			Describe end behavior
The Remainder and	9.4	1	Use the remainder theorem to evaluate polynomials
Factor Theorems			• Use the factor theorem
Roots and Zeros/The	9.5	2	<ul> <li>Apply the rational zero test and synthetic division to find all rational zeros of a polynomial</li> </ul>
Rational Zero Theorem	9.6		Define multiplicity
			<ul> <li>Describe how zeros, factors, and solutions are related</li> </ul>
			• Use the Fundamental Theorem of Algebra to determine the number of solutions of a
			polynomial/draw a connection to the linear factorization theorem
			<ul> <li>Find all (real and complex) zeros of a polynomial</li> </ul>
Graphing Polynomials		1	Graph polynomials
Using the Rational Zero			
Theorem			
Review /Test		2	
	Total	12	
		+3	
		(for additional	
		time/quizzes)	
		15 days	

## UNIT 7 STANDARDS ADDRESSED

## CCSS.MATH.CONTENT.HSA.APR.A.1

Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.

## CCSS.MATH.CONTENT.HSA.SSE.A.1.A

Interpret parts of an expression, such as terms, factors, and coefficients.

## CCSS.MATH.CONTENT.HSA.SSE.A.2

Use the structure of an expression to identify ways to rewrite it. For example, see x4 - y4 as (x2)2 - (y2)2, thus recognizing it as a difference of squares that can be factored as (x2 - y2)(x2 + y2).

#### CCSS.MATH.CONTENT.HSA.APR.B.2

Know and apply the Remainder Theorem: For a polynomial p(x) and a number a, the remainder on division by x - a is p(a), so p(a) = 0 if and only if (x - a) is a factor of p(x).

#### CCSS.MATH.CONTENT.HSF.IF.C.8

Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.

## CCSS.MATH.CONTENT.HSF.IF.C.8.A

Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.

#### CCSS.MATH.CONTENT.HSF.IF.C.7.C

Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.

#### CCSS.MATH.CONTENT.HSA.APR.B.3

Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.

#### CCSS.MATH.CONTENT.HSA.APR.C.4

Prove polynomial identities and use them to describe numerical relationships. For example, the polynomial identity (x2 + y2)2 = (x2 - y2)2 + (2xy)2 can be used to generate Pythagorean triples.

**Unit 8: Trigonometric Ratios/Functions and Graphs** 

Topic	Section In Text	Time Frame	SWBAT
Introduction to Trigonometry	13.1	1	<ul> <li>Evaluate all trigonometric functions (sin, cos, tan, csc, sec, cot) of acute angles with and without the calculator</li> </ul>
Angles and Angle Measure	13.2	2	Extend the unit circle beyond acute angles

Trigonometric Functions of General Angles (+)	13.3	3	<ul> <li>Decide whether angles are coterminal</li> <li>Find coterminals angles</li> <li>Rewrite radian measures as degree and degree as radian</li> <li>Draw angles of rotation</li> <li>Find reference angles</li> <li>Evaluate all trig functions of any angle</li> </ul>
Review/Quest		2	
Graphing Trigonometric Functions	14.1	4	<ul> <li>Define amplitude and period and find each for sin and cos functions.</li> <li>Graph sin and cos functions</li> <li>Graph vertical and horizontal shifts and reflections of sin and cos functions.</li> <li>Write a sin/cos equation given the graph</li> </ul>
Review/Test		2	
	Total	14 +2 (for additional time/ quizzes) 16 days	

## UNIT 8 STANDARDS ADDRESSED

## CCSS.MATH.CONTENT.HSF.TF.A.1

Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.

#### CCSS.MATH.CONTENT.HSF.TF.A.2

Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.

## CCSS.MATH.CONTENT.HSF.TF.B.5

Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.\*

#### CCSS.MATH.CONTENT.HSF.TF.C.8

Prove the Pythagorean identity  $\sin^2(\theta) + \cos^2(\theta) = 1$  and use it to find  $\sin(\theta)$ ,  $\cos(\theta)$ , or  $\tan(\theta)$  given  $\sin(\theta)$ ,  $\cos(\theta)$ , or  $\tan(\theta)$  and the quadrant of the angle.

#### CCSS.MATH.CONTENT.HSF.IF.C.7.E

Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.

## CCSS.MATH.CONTENT.HSF.TF.A.3

(+) Use special triangles to determine geometrically the values of sine, cosine, tangent for  $\pi/3$ ,  $\pi/4$  and  $\pi/6$ , and use the unit circle to express the values of sine, cosine, and tangent for x,  $\pi$  + x, and  $2\pi$  – x in terms of their values for x, where x is any real number.

#### CCSS.MATH.CONTENT.HSF.TF.A.4

(+) Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.

Model periodic phenomena with trigonometric functions.

Unit 9: Trigonometric Inverses, Oblique Triangles, Identities and Equations

Topic	Section	Time	SWBAT
	In Text	Frame	
Inverse Trigonometric	13.4	1.5	Evaluate and graph the inverse of sine and cosine
Functions			Evaluate the inverse of tangent
			Use a calculator to evaluate the inverse of trigonometric functions
The Law of Sines	13.5	2	Solve oblique triangles using law of sines
			<ul> <li>Use law of sines to determine how many solutions exist for an oblique triangle</li> </ul>

			Find the area of an oblique triangle
The Law of Cosines	13.6	1	<ul> <li>Solve oblique triangles using law of cosines</li> <li>Find the area of an oblique triangle using Hero's area formula</li> </ul>
Trigonometric Identities	14.3	3.5	<ul> <li>Recognize and write the fundamental identities</li> <li>Use fundamental trigonometric identities to evaluate, simplify and rewrite trigonometric expressions</li> <li>Verify trigonometric identities</li> </ul>
Solving Trigonometric Equations	14.4	4	<ul> <li>Use algebra to solve trigonometric equations (including quadratic)</li> <li>Use inverse trigonometric functions to solve trigonometric equations</li> </ul>
Review/Test		2	
	Total	14 +2 (for additional time/ quizzes) 16 days	

## UNIT 9 STANDARDS ADDRESSED

## CCSS.MATH.CONTENT.HSF.TF.B.6

(+) Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed.

## CCSS.MATH.CONTENT.HSF.TF.B.7

(+) Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of

the context.\*

Prove and apply trigonometric identities.

## CCSS.MATH.CONTENT.HSF.TF.C.8

Prove the Pythagorean identity  $\sin^2(\theta) + \cos^2(\theta) = 1$  and use it to find  $\sin(\theta)$ ,  $\cos(\theta)$ , or  $\tan(\theta)$  given  $\sin(\theta)$ ,  $\cos(\theta)$ , or  $\tan(\theta)$  and the quadrant of the angle.

Unit 1: Analyzing Equations and Inequalities

	Creating Equations
<ul> <li>Big Ideas: Course Objectives / Content State</li> <li>Number Types: Rational, Irrational, I</li> <li>Solving Equations</li> <li>Solving Absolute Value Equations ar</li> </ul>	ment(s) Integers, Whole, and Natural Numbers
Essential Questions What provocative questions will foster inquiry, understanding, and transfer of learning?	Enduring Understandings What will students understand about the big ideas?
<ul> <li>How do we apply properties of real numbers to simplify expressions and solve equations?</li> <li>Why do absolute value equations usually have two solutions?</li> <li>What are functions and how do we use them?</li> </ul>	<ul> <li>Students will understand that</li> <li>Properties of real numbers help to simplify expressions and make it easier to find the solutions to even the most complicated equations.</li> <li>Absolute value represents the distance from zero and thus, x can have two solutions.</li> <li>Functions are commonly represented in four ways; verbally, numerically, graphically and algebraically.</li> </ul>
Areas of Focus: Proficiencies (Cumulative Progress Indicators)	Examples, Outcomes, Assessments
Students will:	<ul> <li>Instructional Focus (2 weeks):</li> <li>This unit review Algebra I and Foundations skills.</li> <li>Number Types: Students will be able to simplify using order of operations and then classify that number by the highest order that the number falls into.</li> <li>Solving Formulas: This section is geared towards application of mathematics in terms of science topics. Students do not need to be able to factor to solve for a certain variable, but there should be distribution techniques used and combining like terms.</li> <li>Absolute Value Equations/Inequalities: Relate this topic to distance. Example: A person is on the parkway at exit 140 an is 10 miles away from their destination. What exit is he or she at? Students should see that there are two answers and thus absolute value has two answers.</li> </ul>

A.CED.1 Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions.

A.CED.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

A.CED.3 Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.

A.CED.4 Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. For example, rearrange Ohm's law V = IR to highlight resistance R.

CCSS.MATH.CONTENT.HSF.IF.C.7.B Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.

CCSS.MATH.CONTENT.HSF.BF.B.3 Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both • Functions: Students will determine whether relations between two variables are functions. They will also identify and describe the domain and range using interval notation.

#### Assessments:

- 1 sectional quiz
- 1 unit test

## Instructional Strategies:

- Use of graphic organizers to learn properties of real numbers
- Use of venn-diagrams to understand number types
- Use of worksheets and stations to practice skills

Interdisciplinary Connections/Media Literacy Integration/Global Perspectives: The formula below allows students to find the day of the week for which an important date took place

$$w = d + 2m + \left[\frac{3(m+1)}{5}\right] + y + \left[\frac{y}{4}\right] - \left[\frac{y}{100}\right] + \left[\frac{y}{400}\right] + 2$$

d = day of the month

m = monthNote: January and September (9) March (3) February are April (4) October (10) considered the 13th and 14th months of November (11) May (5) the previous year. December (12) June (6) Example: Feb 22, July (7) January (13) 1996 is (14/22/95) August (8) February (14)

y = the year

positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.

Have students research a famous date from another country (suggestion-independence dates, beginning of wars,...) and have students figure out what day of the week these occurred on.

**Technology Integration** 

Show students how to use the STO> button on their TI-83 calculator to help solve complicated substitution problems.

Use Activity Builder by Desmos (sliders) to make observations about how the coefficients "a", "h" and "k" impact the absolute value function.

## Unit 2: Systems of Equations and Inequalities

Reasoning with Equations & Inequalities & Creating Equations

Big Ideas: Course Objectives / Content Statement(s)  Solve systems by graphing, elimination, and substitution  Solve systems with word problems  Solve systems of three variables  Use technology to solve systems	
<ul> <li>Solve systems with word problems</li> <li>Solve systems of three variables</li> </ul>	
Solve systems of three variables	
Use technology to solve systems	
Solve linear programming problems	
Essential Questions Enduring Understandings	
What provocative questions will foster What will students understand about the big ideas?	
inquiry, understanding, and transfer of	
learning?	
<ul> <li>How many solutions can exist given Students will understand that</li> </ul>	
a system of equations?  • A system of equations can have one solution (ie. there is exactly one)	point that
<ul> <li>How can solutions to linear systems each line shares).</li> </ul>	
of equations and inequalities help  • A system of equations can have an infinite number of solutions (ie. the	e linear
with cost effectiveness? functions share all points because the lines share a common slope and	
y-intercept).	
<ul> <li>A system of equations can have no solution (ie. the lines share no cor</li> </ul>	ımon
points because the lines share a common slope but a different y-interc	ept).
Linear programming can be used in making real-life economic decision.	ons

Areas of Focus: Proficiencies	
(Cumulative Progress Indicators)	

## Examples, Outcomes, Assessments

#### Students will:

#### CCSS.MATH.CONTENT.HSA.REI.D.10

Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).

#### CCSS.MATH.CONTENT.HSA.REI.D.11

Explain why the x-coordinates of the points where the graphs of the equations y = f(x) and y = g(x) intersect are the solutions of the equation f(x) = g(x); find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where f(x) and/or g(x) are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.\*

## CCSS.MATH.CONTENT.HSA.REI.D.12

Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.

## Instructional Focus (2 weeks):

• Find the solution(s) given a system of equations.

## Sample Problems:

Systems of Equations

• Solve the system by elimination, substitution and graphically

$$4x - 3y = 18$$

$$3x + y = 7$$

- The school that Lisa attends is selling tickets to the annual talent show. On the first day of ticket sales the school sold 4 senior citizen tickets and 5 student tickets for a total of \$102. The school took in \$126 on the second day by selling 7 senior citizen tickets and 5 student tickets. What is the price each of one senior citizen ticket and one student ticket?
- A plane traveled 580 miles to Ankara and back. The trip there was with the wind. It took 5 hours. The trip back was into the wind. The trip back took 10 hours. Find the speed of the plane in still air and the speed of the wind.

## **Linear Programming**

• A farmer has 25 days to plant cotton and soybeans. The cotton can be planted at a rate of 9 acres per day, and the soybeans at a rate of 12 acres per day. The farm has 275 acres available. If the profit for cotton is \$25 per acre and \$18 per acre, how many of each should be planted to maximize profit?

#### Assessments

- 1 sectional quiz
- 1 unit test

## **Problem Solving Activity**

• One day, while repairing a watch, a watchmaker removed the hour and minute hands. But, she put the hands back on the opposite spindles. When the

customer picked up the watch, the time correctly showed 2:00pm. When is the next time the watch showed the correct time?

- Solve using a table.
- Solve using a system (Hint: Let (D) represent the distance the minute hand moves on the watch and (d) represent the distance the hour hand moves on the watch. Write two equations, one for the wrong watch and one for a correct watch.

## **Technology Integration**

Have students use their graphing calculator to find a solution to a system of equations.

## **Interdisciplinary Connection**

A portion of the subway in Washington DC heads out of the main part of town in the northwesterly direction. It goes under New Hampshire Ave as shown at the right. If the distances are measured in kilometers, the path of the subway can be represented

by the equation y = -2.5x + 2.5 and the path of New Hampshire Ave can be represented by the equation y = x. What are the coordinates of the point at which the subway goes under New Hampshire Ave.

## Media Literacy

Have students research real life scenarios for systems of equations.

## Unit 3: Quadratic Equations and Parabolas

Interpreting Functions & Reasoning with Equations and Inequalities & Seeing Structure in Expressions		
<ul> <li>Big Ideas: Course Objectives / Content States</li> <li>Factor quadratic polynomial expression</li> <li>Find the solutions to a quadratic equal</li> <li>Perform operations with complex numbers</li> <li>Analyze graphs of quadratic functions</li> <li>Essential Questions</li> <li>What provocative questions will foster inquiry, understanding, and transfer of learning?</li> </ul>	ment(s) ons completely tion by graphing, factoring, completing the square and using the quadratic formula nbers	
<ul> <li>How does factoring quadratics help us solve polynomial equations?</li> <li>What does it mean to be a root to a quadratic equation?</li> <li>What are ways we can find roots?</li> <li>How many roots does a quadratic equation have?</li> <li>What is an imaginary number and how do they form the set of complex numbers?</li> <li>What are the advantages of a quadratic function in vertex form? In standard form? In intercept form?</li> <li>How are quadratic functions used to model, analyze and interpret mathematical relationships?</li> </ul>	<ul> <li>Students will understand that</li> <li>All quadratics are not factorable.</li> <li>The methods used to factor a quadratic completely are; gcf, difference of squares, factor/sum (grouping).</li> <li>Roots are solutions to a quadratic equation. They are also known as zeros.</li> <li>A root can be found where the quadratic function crosses the x-axis.</li> <li>A quadratic function can have 0, 1 or 2 real roots. If a function has zero real roots, then the roots to the function are, in fact, imaginary.</li> <li>All quadratic functions are a transformation of the parent function f(x) = x².</li> <li>Quadratic functions are used to maximize profit and minimize cost.</li> </ul>	
Areas of Focus: Proficiencies (Cumulative Progress Indicators)	Examples, Outcomes, Assessments	
CCSS.MATH.CONTENT.HSA.SSE.A.1	Instructional Focus (4 weeks):	

Interpret expressions that represent a quantity in terms of its context.\*

# CCSS.MATH.CONTENT.HSA.SSE.A.1.

Interpret parts of an expression, such as terms, factors, and coefficients.

CCSS.MATH.CONTENT.HSA.SSE.A.2 Use the structure of an expression to identify ways to rewrite it. For example, see x4 - y4 as (x2)2 - (y2)2, thus recognizing it as a difference of squares that can be factored as (x2 - y2)(x2 + y2).

CCSS.MATH.CONTENT.HSF.IF.C.8.A Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.

CCSS.MATH.CONTENT.HSF.IF.C.7.A Graph linear and quadratic functions and show intercepts, maxima, and minima.

CCSS.MATH.CONTENT.HSF.IF.C.9
Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.

• Factor quadratics completely

Sample Problems:

$$3x^2 - 18x$$
$$12x^2 - 22x - 4$$

• Solve a quadratic equation by using square roots, factoring, competing the square or by the quadratic formula.

Sample Problems:

Graph. 
$$f(x) = 2x^2 + 4x - 6$$

Write in vertex form, state the vertex and find the exact and approximate solutions.

Solve by factoring

$$x^2 + 2x - 35 = 0$$

$$64x^2 - 169 = 0$$

Solve by completing the square

$$x^2 - 12x - 10 = 0$$

Solve by the quadratic formula

$$-x^2-6x-1=0$$

Write an quadratic equations with the following roots

$$5 \& -\frac{7}{2}$$

• Complex Numbers

$$2\sqrt{-50}\cdot\frac{1}{8}\sqrt{-2}$$

CCSS.MATH.CONTENT.HSF.BF.B.3 Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.

CCSS.MATH.CONTENT.HSN.CN.A.1 Know there is a complex number has such that i2 = -1, and every complex number has the form a + bi with a and b real.

CCSS.MATH.CONTENT.HSN.CN.A.2 Use the relation i2 = -1 and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.

CCSS.MATH.CONTENT.HSN.CN.C.7 Solve quadratic equations with real coefficients that have complex solutions.

CCSS.MATH.CONTENT.HSN.CN.C.8 (+) Extend polynomial identities to the complex numbers. For example, rewrite  $x^2 + 4$  as (x + 2i)(x - 2i).

# CCSS.MATH.CONTENT.HSN.CN.C.9 (+) Know the Fundamental Theorem of

$$2\sqrt{-18} + 3\sqrt{-2}$$

$$(2-4i)^2$$

$$(1-i^5)(1+i^5)$$

Maximum Profit Example:

The daily revenue R achieved by selling x boxes of candy is figured to be  $R(x) = 9.5x - 0.04x^2$ . How many boxes of candy must the firm sell to maximize revenue? What is the maximum revenue?

#### Assessments

- 2 sectional quizzes
- 1 unit test

## Technology

Use a graphing calculator to check solutions. Show students how to find estimate zeros using upper and lower bounds

## Media Literacy

The game Angry Birds has become a nation wide phenomenon that uses parabolas and projectile motion to win the game. Have students research the questions: What is projectile motion? Are there any other games that use parabolas?

## **Interdisciplinary Connection**

Physics: A ball is thrown up with an initial velocity of 56 feet per sec. The height of the ball t seconds after it is thrown is given by the equation  $h(t) = 56t - 16t^2$ .

- What is the height of the ball after one second?
- What is the maximum height?
- After how many seconds will it return to the ground?

Algebra; show that it is true for quadratic	
polynomials.	

## Unit 4: Functions

# Standard F-IF, F-BF, F-TF & A-REI Interpreting Function, Building Functions, Trigonometric Functions & Reasoning with Equations and Inequalities

Big Ideas: Course Objectives / Content Statement(s)

- Functions
- Domain and Range
- Operations with Functions
- Inverse Functions
- Piecewise Functions
- Absolute Value, Step, Parabolic, Rational, Square Root, and Cube Root Functions
- Intersecting Functions

Essential Questions What provocative questions will foster inquiry, understanding, and transfer of learning?	Enduring Understandings What will students understand about the big ideas?
<ul> <li>What is domain and range? Are there ever any values that cannot be in the domain of a function?</li> <li>How are the properties of functions and functional operations useful?</li> <li>What is vertex form and how can it be applied to functions?</li> </ul>	<ul> <li>Students will understand that</li> <li>Domain refers to x values where as the range refers to the y values. Yes, rational functions and square root functions are the most notable functions that have restricted domains.</li> <li>The properties of functions and function operations are used to model and analyze real-world applications and relationships.</li> <li>Using parent graphs and rules of vertex form, any function can be manipulated to be easily graphed</li> </ul>
Areas of Focus: Proficiencies (Cumulative Progress Indicators)	Examples, Outcomes, Assessments
Students will: CCSS.MATH.CONTENT.HSF.IF.C.7 Graph functions expressed symbolically	Instructional Focus (2 weeks):  • Students should receive an intense exposure to functions.
and show key features of the graph, by hand in simple cases and using technology for more complicated cases.*	Functions: Students should work with operations of functions. They should be able to add, subtract, multiply, divide, simplify and compose functions. They should be able to identify the domain of any function, both algebraically and graphically.

CCSS.MATH.CONTENT.HSF.IF.C.7.B Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.

CCSS.MATH.CONTENT.HSF.BF.B.3 Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.

CCSS.MATH.CONTENT.HSF.BF.B.4 Find inverse functions.

CCSS.MATH.CONTENT.HSF.BF.B.4.A Solve an equation of the form f(x) = c for a simple function f that has an inverse and write an expression for the inverse. For example,  $f(x) = 2 \times 3$  or f(x) = (x+1)/(x-1) for  $x \ne 1$ .

CCSS.MATH.CONTENT.HSF.BF.B.4.B (+) Verify by composition that one function is the inverse of another.

CCSS.MATH.CONTENT.HSA.REI.D.11

Special Types of Functions: Students should use parent graphs to graph special function types. Include Absolute Value, Step, Parabolic, Square Root, Cube Root, and Periodic Functions.

Piecewise Functions: Students should graph linear/quadratic piecewise functions. Expose students to real life scenarios of piecewise functions.

Assessments: 1 unit test

Intersecting Functions: The calculator should be a dominant tool in this section. Students should work with rational, parabolas, lines and other types of polynomials.

Real life problem/Interdisciplinary:

The minimum payment on a credit card is based on the total amount owed. A credit card company uses the following rules: For a bill less than \$10 the entire amount is due. For a bill of at least \$10 but less than \$500, the minimum due is \$10. There is a minimum of \$30 due on a bill of at least \$500 but less than \$1000, a minimum of \$50 due on a bill of at least \$1000, but less than \$1500, and a minimum of \$70 is due on bills \$1500 or more. Find the function f that describes the minimum payment due on a bill of x dollars. Graph the function.

## Technology:

http://www.analyzemath.com/Calculators/Parabola Line.html

http://www.mathsisfun.com/data/function-grapher.php



Explain why the x-coordinates of the points where the graphs of the equations y = f(x) and y = g(x) intersect are the solutions of the equation f(x) = g(x); find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where f(x) and/or g(x) are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.\*

CCSS.MATH.CONTENT.HSF.IF.B.4
For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.\*

CCSS.MATH.CONTENT.HSF.IF.B.5
Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.\*

# Unit 5: Radicals/Rational Exponents The Real & Complex Number System & Reasoning with Equations

Big Ideas: Course Objectives / Content State	ement(s)
Essential Questions What provocative questions will foster inquiry, understanding, and transfer of learning?	Enduring Understandings What will students understand about the big ideas?
<ul> <li>Why is √125=5√5?</li> <li>Why is it necessary for roots to "match" in order to add or subtract radicals?</li> <li>Why can we rationalize that √x = x<sup>1/2</sup>?</li> <li>How do you find the solutions to a radical equation? What are extraneous solutions?</li> </ul>	<ul> <li>Students will understand that</li> <li>Both √125and5√5 have the same approximate decimal value. They will then, notice that 125 is equivalent to the perfect square, 25, times the non perfect square, 5, thus making the two radical expression equivalent.</li> <li>Just like variables must be alike to combine like terms, so must radicals as they too are two numerical values being multiplied together. They 3x+4y-2x+y  &amp; will see the relationship between 3√2+4√3-2√2+√y</li> <li>Power rules can be applied to radicals. Since √x² = x and(x²)¹² = x, a ½ power must be the same operation as a square root.</li> <li>Radical equations can be solved by isolating the radical and squaring both sides of the equation. This process can introduce extraneous solutions.</li> </ul>

# Areas of Focus: Proficiencies (Cumulative Progress Indicators)

## Examples, Outcomes, Assessments

#### Students will:

CCSS.MATH.CONTENT.HSA.REI.A.2 Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.

## CCSS.MATH.CONTENT.HSN.RN.A.1

Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. For example, we define  $5^{1/3}$  to be the cube root of 5 because we want  $(5^{1/3})^3 = 5^{(1/3)3}$  to hold, so  $(5^{1/3})^3$  must equal 5.

## CCSS.MATH.CONTENT.HSN.RN.A.2

Rewrite expressions involving radicals and rational exponents using the properties of exponents.

## CCSS.MATH.CONTENT.HSN.RN.B.3

Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.

## Instructional Focus (2 weeks):

• Simplify expressions with radicals and rational roots.

## Sample Problems:

• Simplifying radicals

$$- (\sqrt{10} + \sqrt{6})(\sqrt{30} - \sqrt{18})$$

$$- \sqrt{\frac{1}{18}}$$

• Rational Exponents

$$(-64)^{\frac{2}{3}}$$

Solving radical expressions

$$5 + \sqrt{x+4} = 1i$$
$$-(x+6)^{\frac{1}{3}} = 2$$

## Assessments

- 1 sectional quiz
- 1 unit test

## Instructional Strategies:

• To help students see the difference between  $\sqrt{x^2}$  and  $(\sqrt{x})^2$ , have students warm up by filling in the chart below.

CCSS.MATH.CONTENT.HSF.IF.C.7.B
Graph square root, cube root, and
piecewise-defined functions, including
step functions and absolute value

functions.

λ	$\left(\sqrt{\chi^2}\right)$	$\left(\sqrt{x}\right)^{z}$	$\left(\sqrt[3]{x^3}\right)$	$(\sqrt[3]{x})^{\xi}$
1				
-1				
64				·
-64				

• Police officers use formulas to investigate crime. One formula that is used often deals with a driver's speed.  $s = 2\sqrt{5}$  is a formula that relates skid marks to speed traveled by a driver. A police officer was investigating a crime in which the driver claimed that she was only traveling 40 miles/hr. The police officer measured the skid marks to be 120ft. Was she lying?

**Interdisciplinary Connection** 

Physics: Find the time that it takes a pendulum to complete a swing it its length

is 10 inches. Use the formula  $T = 2\pi \sqrt{\frac{L}{38^2}}$  where T represents time in second and L represents the length of the pendulum in inches.

Media Literacy

Have students research the Mandelbrot set and explain how imaginary numbers are used to create these fractals.

**Technology Integration** 

 $\underline{http://www.youtube.com/watch?v=gEw8xpb1aRA}$ 

Unit 6: Exponential and Logarithmic Functions

Standard F-LE & F-BF Linear, Quadratic, and Exponential Models & Building Functions		
Big Ideas: Course Objectives / Content State     Graphs of Exponential and Logarithm     Logarithms and their Properties     Natural Logarithms	ment(s)	
<ul> <li>Word Problems with Logarithms</li> </ul>		
Essential Questions What provocative questions will foster inquiry, understanding, and transfer of learning?	Enduring Understandings What will students understand about the big ideas?	
<ul> <li>How are exponents and logarithms related?</li> <li>What is a logarithm and what is it used for?</li> <li>How are properties of exponents applied to logarithms?</li> <li>What is the importance of logarithms in real life data?</li> </ul>	<ul> <li>Students will understand that</li> <li>The exponential and logarithmic functions are inverses.</li> <li>A logarithm is a function that allows one to find the solution to a variable exponential value.</li> <li>The summation of logarithms is derived from multiplying values with like bases, subtraction from division, and multiplication from power rules.</li> <li>Since logarithms can solve for exponential variables, it is often used to solve for rate and time in Interest problems.</li> </ul>	
Areas of Focus: Proficiencies (Cumulative Progress Indicators)	Examples, Outcomes, Assessments	
Students will:  CCSS.MATH.CONTENT.HSA.SSE.A.1.B	Instructional Focus (3 weeks):  • Exploring Logarithms.	
Interpret complicated expressions by viewing one or more of their parts as a single entity. For example, interpret P(1+r)n as the product of P and a factor not depending on P.	Sample Problems:  Evaluate.  1	
CCSS.MATH.CONTENT.HSF.IF.C.7.E	$\log_7 \frac{1}{4^{\frac{1}{2}}}$	

Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.

CCSS.MATH.CONTENT.HSF.IF.C.8.B

Use the properties of exponents to interpret expressions for exponential functions. For example, identify percent rate of change in functions such as  $y = (1.02)^t$ ,  $y = (0.97)^t$ ,  $y = (1.01)12^t$ ,  $y = (1.2)^t/10$ , and classify them as representing exponential growth or decay.

CCSS.MATH.CONTENT.HSF.BF.B.5

(+) Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.

CCSS.MATH.CONTENT.HSF.LE.A.4 For exponential models, express as a logarithm the solution to abct = d where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using

technology.

$$\log_4 x = \frac{3}{2}$$

$$2\log x + \log 3 = \log 27$$

$$\log_2 6 - \log_2 (x+4) = 3$$

$$-3.11^{4}-6=-22$$

$$\ln 5x + \ln x = 7$$

Assessments

- 2 sectional quizzes
- 1 unit test

Technology

Use a graphing calculator to relate  $f(x) = 10^{\circ} \& f(x) = \log x$ 

What relationship is seen between the two functions?

Interdisciplinary Connection

How many days will it a take a culture of bacteria to increase from 2000 to 50,000 if the growth rate per day is 93.2%?

Carl plans to invest \$500 at and interest rate of 8.25%, compounded continuously. How long will it take him to have \$2000 in his account?

### Media Literacy/Global Studies

Have students explore the internet to the change in interest rates over the past 10 years, both in interest charged and interest earned. How can this effect an investment of \$10,000? Now have students look at the current interest rates around the world and compare them with our... Are they better or worse? Be sure to convert all units to dollars during the comparisons.

### Unit 7: Exploring Polynomial Functions

Arithmetic with Polynomials and Rational Expressions & Seeing Structure in Expressions & Interpreting Functions & Building Functions

Big Ideas: Course Objectives / Content Statement(s)

- Polynomial Operations
- The Remainder and Factor Theorem
- Graphing Polynomials

- What is synthetic division and why is it useful? Why must we include zeros for missing powers?
- What is the relationship between a polynomial and it's factors?
- What are roots to polynomial equations?
- What types of roots exist for polynomial equations?
- How are factors, zeros and x-intercepts related? How are factors and roots related?
- What are the differences between even and odd polynomials graphically?
- How do you compose functions?

Enduring Understandings What will students understand about the big ideas?

Students will understand that...

- The properties of a real number can be used to multiply a monomial by a polynomial, simplify the product of binomials/trinomials.
- Synthetic division is a quick way to divide two polynomials without using long division. We must include zeros for missing powers in synthetic division because each position represents a power of the variable. If you do not put in zeros, then an expression such as  $x^4 + x^2 + 2$  will be written as 1 1 2, but this represents  $x^2 + x + 2$ . We would, instead, enter 1 0 0 1 2.
- Factoring is the process of breaking down a polynomial into the multiplication of two or more polynomial or monomial expressions. Similar to a number that can be factored into it's prime factors, a polynomial can be factored into it's composite terms.
- Roots are where the polynomial equation equals zero. Real zeros cross or touch the x-axis.
- There are rational, irrational, and imaginary roots in polynomial equations. Number of real and imaginary roots always equals the degree of the polynomial.
- If (x-a) is a factor of a polynomial, then the polynomial has value zero when x = a. If a is a real number, then the graph of the polynomial has (a, 0) as an x-intercept.

•	What are the characteristics of
	a polynomial function?

- How does left/right and even/odd behavior help in graphing polynomial functions?
- A polynomial function has distinguishing "behaviors'. You can look at its algebraic form and know something about its graph or look at its graph and know something about its algebraic form.
- In an even polynomial, the end behaviors of the polynomial either both increase or both decrease, whereas in an odd polynomial, one will increase while the other will decrease.

# Areas of Focus: Proficiencies (Cumulative Progress Indicators)

### Students will:

# CCSS.MATH.CONTENT.HSA.APR. A.1

Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.

# CCSS.MATH.CONTENT.HSA.SSE. A.1.A

Interpret parts of an expression, such as terms, factors, and coefficients.

# CCSS.MATH.CONTENT.HSA.SSE. A.2

Use the structure of an expression to identify ways to rewrite it. For example, see x4 - y4 as (x2)2 - (y2)2, thus recognizing it as a difference of squares that can be factored as (x2 - y2)(x2 + y2).

### Examples, Outcomes, Assessments

### Instructional Focus (3 weeks):

• Add, subtract, multiply, divide and factor polynomials.

### Sample Problems:

• Polynomial Multiplication

$$(2a-1)(8a-5)$$
  
 $(4p-1)^2$ 

$$(6n^2-6n-5)(7n^2+6n-5)$$

Factoring

GCF: 
$$x + x^2y + x^3y^2$$

Grouping: 
$$12p^3 - 21p^2 + 28p - 4$$

$$a = 1 : x^2 + 5x - 6$$

$$a > 1$$
:  $6x^2 + 5x - 6$ 

difference of squares :  $4a^2 - 2!$ 

sum of cubes:  $216 + x^3$ 

• Polynomial Division

Long Division:

# CCSS.MATH.CONTENT.HSA.APR. B.2

Know and apply the Remainder Theorem: For a polynomial p(x) and a number a, the remainder on division by x - a is p(a), so p(a) = 0 if and only if (x - a) is a factor of p(x).

### CCSS.MATH.CONTENT.HSF.IF.C.8

Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.

# CCSS.MATH.CONTENT.HSF.IF.C.8

Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.

# CCSS.MATH.CONTENT.HSF.IF.C.7

Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.

# CCSS.MATH.CONTENT.HSA.APR. B.3

Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough

$$(2x^3 + 5x^2 - 2x - 1) \div (2x - 3)$$

Synthetic Division(Finding remaining factors):

$$(x^3-13x^2+24x+108)$$
÷  $(x+2)$ 

Examining polynomials.

Sample Problems:

Find the remaining factors

$$x^4 + 14x^3 + 51x^2 + 54x$$
;  $x + 9$ 

Find the zeros to the polynomial. Then sketch a graph.

$$f(x) = x^4 + 2x^3 - 8x^2 - 18x - 9$$

Sketch a possible graph of

 $y = -2(x-2)^2(3x+4)^3(x^2+1)$ . How many x-intercepts does the function have?

Use quadratic techniques to solve.

$$x^4 + x^2 - 20 = 0$$
  
 $x^{\frac{1}{2}} - 4x^{\frac{1}{4}} + 3 = 0$ 

Assessments

- 3 sectional quizzes
- 1 unit test

**Interdisciplinary Connection** 

Connect the concept of Punnett Squares from biology to polynomial multiplication.

graph of the function defined by the polynomial.

CCSS.MATH.CONTENT.HSA.APR. C.4

Prove polynomial identities and use them to describe numerical relationships. For example, the polynomial identity (x2 + y2)2 = (x2 - y2)2 + (2xy)2 can be used to generate Pythagorean triples. Media Literacy

http://www.youtube.com/user/WSHSmath#p/u/2/OFSrINhfNsQ

Show students the factoring video and assign them the task of creating their own video about a concept in this chapter.

# Unit 8: Trigonometric Ratios/Functions and Graphs Trigonometric Functions & Interpreting Functions & Building Functions

Big Ideas: Course Objectives / Content Sta	atement(s):
<ul><li>Exploring Trigonometric Functions</li><li>Graphs of sine and cosine</li></ul>	
Gruphs of sine and cosine	
Essential Questions	Enduring Understandings
What provocative questions will foster	What will students understand about the big ideas?
inquiry, understanding, and transfer of	
learning?	
<ul> <li>How are the six basic trig</li> </ul>	Students will understand that
functions related?	<ul> <li>There are six trigonometric functions which are ratios of sides of right</li> </ul>
<ul> <li>How are radian and degree</li> </ul>	triangles.
measures used in real-world	<ul> <li>Angles can be measured in radians and degrees.</li> </ul>
settings?	<ul> <li>Trigonometric functions can be extended to all real numbers, using the</li> </ul>
<ul> <li>What is the unit circle and how</li> </ul>	unit circle (CCSS HSF.TF.A.2)
can it help to evaluate	• The cosine function corresponds with the x-coordinate of the point where
trigonometric functions quickly?	the terminal side of the angle intersects the unit circle. The sine function
• What is a periodic function?	corresponds with the y-coordinate of the point where the terminal side of
How do amplitudes, periods,	the angle intersects the unit circle.
phase shifts, vertical shifts and	Trigonometric functions are periodic. Their graphs are unique because
cofunctions relate to the graphs of	they are continuous curves that repeat themselves.
translated sine and cosine	• Changes to the algebraic equation of a function cause predictable changes
functions?	to the function's graph.
Areas of Focus: Proficiencies	Examples, Outcomes, Assessments
(Cumulative Progress Indicators)	Estamples, Gateonies, Lissessinents
Students will:	Instructional Focus:
	An introduction to trigonometry
CCSS.MATH.CONTENT.HSF.TF.A.1	Angles and angle measure
Understand radian measure of an angle as	Trigonometric Functions of General angles
the length of the arc on the unit circle	Graphing Trigonometric Functions

subtended by the angle.

CCSS.MATH.CONTENT.HSF.TF.A.2 Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.

CCSS.MATH.CONTENT.HSF.TF.B.5 Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.\*

CCSS.MATH.CONTENT.HSF.TF.C.8 Prove the Pythagorean identity  $\sin^2(\theta) + \cos^2(\theta) = 1$  and use it to find  $\sin(\theta)$ ,  $\cos(\theta)$ , or  $\tan(\theta)$  given  $\sin(\theta)$ ,  $\cos(\theta)$ , or  $\tan(\theta)$  and the quadrant of the angle.

CCSS.MATH.CONTENT.HSF.IF.C.7.E Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.

CCSS.MATH.CONTENT.HSF.TF.A.3 (+) Use special triangles to determine geometrically the values of sine, cosine, tangent for  $\pi/3$ ,  $\pi/4$  and  $\pi/6$ , and use the unit circle to express the values of sine, cosine, and tangent for x,  $\pi$  + x, and  $2\pi$  – x in terms of their values for x, where x is any real number.

### Assessments:

- 2 sectional quizzes
- 1 unit test

Have students create and use diagrams to model examples of the terms: arc, standard position, radian, and coterminal.

### Sample Assessments:

- Have students create and use diagrams to model examples of the terms: arc, standard position, radian, and coterminal angles. Display these in the classroom as a visual.
- Students create a tessellation using their knowledge of angles gained from this unit.

### Instructional Strategies:

**Interdisciplinary Connections** 

• PHYSICS: Have students refer to a physics text and look through the section on waves, especially light and sound. They should focus on the meanings of period and amplitude

### **Technology Integration**

• Students use their graphing calculators or online graphing applets to visualize the effect of changing the period and amplitude of a trigonometric graph vs. its parent function.

Global Perspectives/Culturally Responsive Teaching

• Sundials use a shadow falling on a calibrated scale to tell time. In some primitive regions of Egypt, sundials are still used to tell time.

\*(See Chapters 13 and 14 for extensions of these ideas)

# CCSS.MATH.CONTENT.HSF.TF.A.4 (+) Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions. Model periodic phenomena with trigonometric functions.

## Unit 9: Trigonometric Inverses, Oblique Triangles, Identities and Equations

Trigonometric Functions & Interpreting Functions & Building Functions		
Big Ideas: Course Objectives / Content Statement(s):		
<ul> <li>Inverse Trigonometry</li> </ul>		
<ul> <li>Oblique Triangles</li> </ul>		
<ul> <li>Trigonometric Identities and Equation</li> </ul>	ons	
Essential Questions	Enduring Understandings	
What provocative questions will foster	What will students understand about the big ideas?	
inquiry, understanding, and transfer of		
learning?		
<ul> <li>How do the graphs of the inverse trig functions relate to the parent graphs of the trig function?</li> <li>How do you find the missing information from a given oblique triangle? How do you use oblique triangles to solve real world situations?</li> <li>How can algebraic operations be used to simplify trigonometric expressions and verify trigonometric identities?</li> <li>How do you solve trig equations?</li> </ul>	<ul> <li>Students will understand that</li> <li>Some trig functions must have their domain restricted so that you can find an inverse of the function.</li> <li>If triangles are not right, most triangles can be solved using the Law of Sines or the Law of Cosines. The Law of Sines and the Law of Cosines can be used to solve many application problems, including area.</li> <li>The basis of trigonometric identities comes from both the unit circle and the Pythagorean Theorem. Manipulation, substitution, and reduction can simplify complex trigonometric expressions.</li> <li>Using algebraic techniques and/or inverse trig functions can solve trigonometric equations.</li> </ul>	

Areas of Focus: Proficiencies (Cumulative Progress Indicators)	Examples, Outcomes, Assessments	
Students will:  CCSS.MATH.CONTENT.HSF.TF.B.6  (+) Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed.  CCSS.MATH.CONTENT.HSF.TF.B.7  (+) Use inverse functions to solve trigonometric equations that arise in	Instructional Focus:      Use inverse trigonometric to find angle measurements     Solve oblique triangles     Verify identities     Solve trigonometric equations  Assessments:     1 sectional quiz     1 unit test  Sample problems:	
modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.*  Prove and apply trigonometric identities.  CCSS.MATH.CONTENT.HSF.TF.C.8  Prove the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$ and use it to find $\sin(\theta)$ , $\cos(\theta)$ , or $\tan(\theta)$ given $\sin(\theta)$ , $\cos(\theta)$ , or $\tan(\theta)$ and the quadrant of the angle.	<ul> <li>Have students use a right triangle to solve problems of the type: sin sin (3/4)</li> <li>Use factoring techniques to simplify trigonometric expressions: tan²x - tan²xsin²x</li> <li>sec²x-1/sec sec x - 1</li> <li>Have students solve trigonometric equations that have extraneous solutions. Encourage them to check solutions by hand and by calculator.</li> <li>Sample Assessments:</li> <li>Group Activity: Create and give students a triangle problem where there are two possible sets of answers. Have a discussion on the solutions the groups arrive at.</li> <li>Instructional Strategies:         <ul> <li>Interdisciplinary Connections</li> <li>MUSIC: Musicians – tuning instruments involves sounds waves which can be represented with trigonometric equations</li> </ul> </li> <li>Technology Integration:</li> </ul>	

•	Students use their graphing calculators or online graphing applets to
	compare and contrast the inverse trig graphs and their transformations

• Students can use a graphing utility to verify equivalent expressions

### Global Perspectives/Culturally Responsive Teaching

• Students should research Hipparchus, other scientists and mathematicians to learn about their contributions to trigonometry.

Supports for English Language Learners			
Sensory Supports	Graphic Supports	Interactive Supports	
Real life objects	Charts	In pairs or partners	
Manipulatives	Graphic Organizers	In triands or small groups	
Pictures	Tables	In a whole group	
Illustrations, diagrams & drawings	Graphs	Using cooperative group	
Magazines & Newspapers	Timelines	Structures	
Physical activities	Number lines	With the Internet / Software	
Videos & Film		In the home language	
Broadcasts		With mentors	
Models & Figures			

Intervention Strategies		
Accommodations	Interventions	Modifications
Allow for verbal responses	Multi-sensory techniques	Modified tasks/expectations

Repeat/confirm directions	Increase task structure (e.g. directions, checks for understanding, feedback	Differentiated materials
Permit response provided via computer or electronic device	Increase opportunities to engage in active academic responding	Individualized assessment tools based on student need
Audio Books	Utilize pre reading strategies and activities previews, anticipatory guides, and semantic mapping	Modified assessment grading

### **Career-Ready Practices**

**CRP1**: Act as a responsible and contributing citizen and employee.

CRP2: Apply appropriate academic and technical skills.

CRP3: Attend to personal health and financial well-being.

**CRP4**: Communicate clearly and effectively and with reason.

CRP5: Consider the environmental, social and economic impacts of decisions.

**CRP6**: Demonstrate creativity and innovation.

**CRP7**: Employ valid and reliable research strategies.

**CRP8**: Utilize critical thinking to make sense of problems and persevere in solving them.

CRP9: Model integrity, ethical leadership and effective management.

**CRP10**: Plan education and career paths aligned to personal goals.

**CRP11**:. Use technology to enhance productivity.

**CRP12**: Work productively in teams while using cultural global competence.

### **Summit Public Schools**

Summit, New Jersey

### **Curricular Addendum**

### **Career-Ready Practices**

**CRP1**: Act as a responsible and contributing citizen and employee.

CRP2: Apply appropriate academic and technical skills.

CRP3: Attend to personal health and financial well-being.

**CRP4**: Communicate clearly and effectively and with reason.

**CRP5**: Consider the environmental, social and economic impacts of decisions.

**CRP6**: Demonstrate creativity and innovation.

CRP7: Employ valid and reliable research strategies.

**CRP8**: Utilize critical thinking to make sense of problems and persevere in solving them.

**CRP9**: Model integrity, ethical leadership and effective management.

CRP10: Plan education and career paths aligned to personal goals.

**CRP11**:. Use technology to enhance productivity.

**CRP12**: Work productively in teams while using cultural global competence.

### **Interdisciplinary Connections**

- Close Reading of works of art, music lyrics, videos, and advertisements
- Use <u>Standards for Mathematical Practice</u> and <u>Cross-Cutting Concepts</u> in science to support debate/inquiry across thinking processes

### **Technology Integration**

### Ongoing:

- Listen to books on CDs, Playaways, videos, or podcasts if available.
- Use document camera or overhead projector for shared reading of texts.

### Other:

- Use Microsoft Word, Inspiration, or SmartBoard Notebook software to write the words from their word sorts
- Use available technology to create concept maps of unit learning.

# Instructional Strategies: Supports for English Language Learners:

Sensory Supports	Graphic Supports	Interactive Supports
Real-life objects (realia)	Charts	In pairs or partners
Manipulatives	Graphic organizers	In triads or small groups
Pictures & photographs	Tables	In a whole group
Illustrations, diagrams, & drawings	Graphs	Using cooperative group
Magazines & newspapers	Timelines	structures
Physical activities	Number lines	With the Internet (websites) o
Videos & films		software programs  In the home language  With mentors
Broadcasts		
Models & figures		

from https://wida.wisc.edu

### **Media Literacy Integration**

 Use multiple forms of print media (including books, illustrations/photographs/artwork, video clips, commercials, podcasts, audiobooks, Playaways, newspapers, magazines) to practice reading and comprehension skills.

### **Global Perspectives**

• The Global Learning Resource Library

### **Differentiation Strategies:**

Accommodations	Interventions	Modifications
Allow for verbal responses	Multi-sensory techniques	Modified tasks/ expectations
Repeat/confirm directions	Increase task structure (e.g., directions, checks for understanding, feedback)	Differentiated materials
Permit response provided via computer or electronic device	Increase opportunities to engage in active academic responding (e.g., writing, reading aloud, answering questions in class)	Individualized assessment tools based on student need
Audio Books	Utilize prereading strategies and activities: previews, anticipatory guides, and semantic mapping	Modified assessment grading