Summit Public Schools

Summit, New Jersey

Grade Level 9 / Content Area: Mathematics

Length of Course: Full Academic Year

Curriculum: Honors Geometry

Developed by Alicia Lyle

Submitted August 2019

Curriculum

Course Description:

This advanced course in geometry is based on Euclidean geometry and includes congruence, similarity, parallelism, perpendicularity, transformations, area, and volume. Coordinates and geometric constructions are integrated throughout the course. Students will construct two column, paragraph, and indirect proofs throughout much of the first semester using definitions and proved theorems. Students are expected to use scientific calculators, and will use additional technology such as geogebra at appropriate times. Students will have the opportunity to explore mathematical modeling and application throughout the entire course.

* * *

UNIT 1- INTRODUCTION TO GEOMETRY

Anchor Standard: G-CO: Congruence Students will understand congruence in terms of geometric figures that share size and shape.	
Big Ideas: *Course Objectives/Content Statement(s)* • Students will learn the nuances of geometric symbols and language. • Students will practice logic and deductive reasoning as it applies to geometric proofs. • Students will make and interpret geometric constructions to solve problems.	
Essential Questions *What provocative questions will foster inquiry, understanding, and transfer of learning?*	**Enduring Understandings** *What will students understand about the big ideas?*
	Students will understand that…
What are the defining characteristics of basic geometric figures?	Basic geometric figures are defined by points, lines, and planes. Proper notation is required when naming parts of geometric figures.
How are two-column proofs used in justifying properties or theorems?	The first column in a two-column proof lists consecutive statements that lead to a conclusion. The second column provides justification for each statement.

How is logic used to prove or explain mathematical statements?	Most statements can be written conditionally, i.e. "if p, then q". The negation, converse, inverse, and contrapositive are formed using varied forms and orders of p and q. Once a statement is written conditionally, it can be structured in a two-column proof.
Areas of Focus: Proficiencies (Progress Indicators)	**Examples, Outcomes, Assessments**
Students will: G-CO-1: Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc. G-CO-12: Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.) G-CO-13: Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle. G-GPE-6: Find the point on a directed line segment between two given points that partitions the segment in a given ratio.	**Instructional Focus** 1. Recognize and identify points, lines, segments, rays, angles, triangles, congruent angles and segments, collinear and noncollinear points, betweenness, conditional statements, the negation, converse, inverse and contrapositive of a statement, and bisectors and trisectors of segments and angles. 2. Measure segments and angles. 3. Classify angles and name the parts of a degree. 4. Apply the triangle-inequality principle. 5. Correctly interpret geometric diagrams 6. Understand the characteristics of theorems and how they can be used in proofs. **Sample Assessments:** EX: The measures of three angles of a triangle are in the ratio 1:2:3. Find the measure of each angle, then classify the triangle by its angles. EX: If six points are represented on a sheet of paper in such a way that any four of them are noncollinear, what are the minimum and maximum number of lines determined?

EX: Draw a diagram in which $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ interest at *E* but in which $\angle AEC$ does not appear to be congruent to $\angle DEB$

EX: The perimeter of a rectangle is 20cm. If the rectangle's length is less than 4, what is the range of possible values of its width?

Projects/Post Assessment

Students can do an independent project where they use the free online software geogebra to do geometric constructions.

Instructional Strategies

- Interdisciplinary Connections

Students can read excerpts from The Trisectors, a book written by Underwood Dudley that is a collection of published proofs on trisecting an angle that are in fact incorrect.

- **Technology Integration**

Students can use Geogebra to trisect an angle

- **Media Literacy Integration**

Students can research and analyze works of art from different centuries that show points, lines, and/or planes. For example, the Dutch artist Mondrian is famous for his geometric and rectangular work.

- **Global Perspectives**

Students can research and recreate the origami proof of the trisection of an angle as developed by Italian-Japanese mathematician Humiaka Huzita in the late 20th century.

UNIT 2 – BASIC CONCEPTS AND PROOFS

<table>
<tr><td colspan="2">Anchor Standard: G-CO: Congruence

Students will prove geometric theorems.</td></tr>
<tr><td colspan="2">Big Ideas: Course Objectives/Content Statement(s)
Students will prove geometric theorems involving segment length and angle measure using two column and paragraph proofs.</td></tr>
<tr><td>Essential Questions
What provocative questions will foster inquiry, understanding, and transfer of learning?</td><td>Enduring Understandings
What will students understand about the big ideas?</td></tr>
<tr><td>How can angles or pairs of angles be defined once two lines, rays, or line segments intersect?

How are segments and angles and their properties based on measurement used in proof writing?</td><td>Students will understand that…

Angles are classified by their measure: acute for angles measuring less than 90˚, obtuse for angles measuring more than 90˚, right if an angle measures exactly 90˚, and straight for angles that measure exactly 180˚. A pair of angles whose measures add to 90˚ are called complementary; supplementary if the measures add to 180˚. Two congruent angles are angles with the same measure. Perpendicular lines form four right angles. Vertical angles are formed by two intersecting lines; vertical angles have a common vertex yet do not share sides.

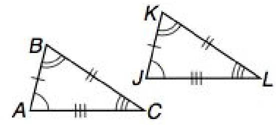
The addition, subtraction, multiplication, division, substitution, and transitive properties can be applied using segments and their lengths and angles and their measures.</td></tr>
<tr><td>Areas of Focus: Proficiencies
(Progress Indicators)</td><td>Examples, Outcomes, Assessments</td></tr>
<tr><td>Students will:</td><td>Instructional Focus
Understand the concept of perpendicularity.</td></tr>
</table>

G-CO-9: Prove theorems about lines and angles. *Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints*	2. Recognize complementary and supplementary angles, opposite rays, and vertical angles. 3. Prove angles congruent. 4. Apply the addition, subtraction, multiplication, division, substitution, and transitive properties of segments and angles. **Sample Assessments:** EX: Given square *BDCE*. Segment *AC* contains point *B* such that *B* is between *A* and *C*. If $\overrightarrow{BC}$ bisects $\angle DBE$, prove $\angle ABD \cong \angle ABE$. EX: Given triangle *ADC* and point *B* such that *B* is between *A* and *C*. If $\angle A$ is complementary to $\angle ADB$, $\angle C$ is complementary to $\angle CDB$, and $\overrightarrow{DB}$ bisects $\angle ADC$, prove $\angle A \cong \angle C$. EX: Consider quadrilateral *ABDC* with $\overline{PBDR}$. Given $\overline{AB} \perp \overline{PR}$ and $\overline{AB} \cong \overline{CD}$, show $\overline{CD} \perp \overline{PR}$. **Projects/Post Assessment** Students can work in groups to solve a complex multi-step proof using the theorems from the chapter. **Instructional Strategies** • **Interdisciplinary Connections** During this introductory unit on proof writing, students can relate the structure of a two-column proof to the structure of an argumentative or persuasive essay.

	• **Technology Integration** Students can prove the angle addition theorem using geogebra. • **Media Literacy Integration** Students can create short video presentations to illustrate angle vocabulary. The videos can be uploaded to a class website or www.youtube.com. • **Global Perspectives** Research James L. Gould's theory about bees and angles, as well as the controversy that surround his ideas.

UNIT 3 – CONGRUENT TRIANGLES

Anchor Standard: G-CO: Congruence Students should be able to prove triangles congruent. Once triangles are proved congruent, students should also be able to apply the principle stating corresponding parts of congruent triangles are congruent, also known as CPCTC.	
Big Ideas: *Course Objectives/Content Statement(s)* • Students will understand congruence in terms of rigid motion. • Students will prove pairs of triangles congruent • Students will prove geometric theorems and other true statements involving congruent triangles.	
Essential Questions *What provocative questions will foster inquiry, understanding, and transfer of learning?*	**Enduring Understandings** *What will students understand about the big ideas?*

What are congruent figures? How can two triangles be proved congruent? If two triangles are congruent, how can additional congruency statements be developed?	**Students will understand that…** Congruent figures are figures that have exactly the same size and shape. All pairs of corresponding parts are congruent. There are three postulates (ASA, SAS, and SSS) and one theorem (AAS) that can be used to prove triangles congruent. The CPCTC principle states corresponding parts of congruent triangles are congruent.
Areas of Focus: Proficiencies (Progress Indicators)	**Examples, Outcomes, Assessments**
Students will: G-CO-7: Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent. G-CO-8: Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions. G-CO-10: Prove theorems about triangles. *Theorems include: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point*	**Instructional Focus:** 1. Decide if two figures are congruent. 2. Identify corresponding parts of congruent figures. 3. Given two congruent figures, solve for unknown measures. 4. Prove two triangles are congruent using postulates SAS, SSS, ASA, AAS and HL. 5. Prove additional congruencies using CPCTC. **Sample Assessments:** EX: Decide if the figures are congruent. If so, write a congruency statement. 

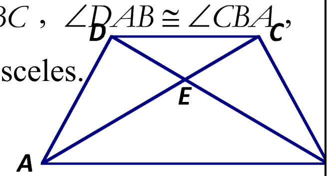
EX: Given $\Delta BIG \cong \Delta CAT$, $IB = 11$,

$AT = 2x - 4y$, $BG = x + y$,

$CA = 4x + y$, $IG = 10$. Find CT.

EX: Given $\overline{AD} \cong \overline{BC}$, $\angle DAB \cong \angle CBA$,

prove ΔABE is isosceles.



Projects/Post Assessment
Students will make their own proof problem, complete with a diagram, given information, and what needs to be proven. They will also provide at least two unique solutions to their proof.

Instructional Strategies

- **Interdisciplinary Connections**

The properties of congruent triangles can be applied to the real world.

EX: Kate and Jaclyn wished to find the distance from N on one side of a lake to P on the other side. They put stakes at N, P, and T, then extended PT to S, making sure that PT was congruent to TS. They followed a similar process in extending NT to R. They then measured SR and found is to be 70m long. They concluded that NP was 70m. Prove that they were correct.

- **Technology Integration**

Students can use geogebra to make the diagrams for proofs.

- **Media Literacy Integration**

	Students can research (online and in text) proofs that have more than one solution. Students can then determine which solutions are acceptable based on the theorems that students have learned through this point in the curriculum. • **Global Perspectives** The properties of congruent triangles are used throughout the world in the construction of bridges, as one example.

UNIT 4 – LINES IN THE PLANE

Anchor Standard: G-GPE: Expressing Geometric Properties with Equations Students should be able to use the correspondence between numerical coordinates and geometric points as it allows methods from algebra to be applied to geometry and vice versa.	
Big Ideas: *Course Objectives/Content Statement(s)* • Students will use coordinates to prove simple geometric theorems involving lines algebraically.	
Essential Questions *What provocative questions will foster inquiry, understanding, and transfer of learning?*	**Enduring Understandings** *What will students understand about the big ideas?*
	Students will understand that…
How is slope defined?	The slope of a line is defined by the ratio of the change in y-coordinates to the change in x-coordinates of two points. Slope can be positive, negative, zero, or undefined.
What must be true of slopes of parallel and perpendicular lines?	Parallel lines have equal slopes and all vertical lines are parallel. Perpendicular lines have opposite reciprocal slopes. Horizontal and vertical lines are perpendicular.

Areas of Focus: Proficiencies (Progress Indicators)	Examples, Outcomes, Assessments
Students will: G-GPE-5: Prove the slope criteria for parallel and perpendicular lines.	**Instructional Focus:** 1. Apply the midpoint formula. 2. Recognize the relationship between equidistance and perpendicular bisection. 3. Recognize planes, transversals, and parallel lines. 4. Understand the concept of slope. 5. Relate the slope of a line to its orientation in the coordinate plane. 6. Recognize the relationships between the slopes of parallel and perpendicular lines. **Sample Assessments:** EX: $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ contain points $A(3, -4)$, $B(0, 0)$, $C(1, 2)$, and $D(4, -2)$. Prove algebraically that lines $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ are parallel, perpendicular, or neither. EX: Prove that a point on a perpendicular bisector of a line segment is equidistance from the endpoints of the bisected line segment. **Projects/Post Assessment** Students can identify where in their homes they see different types of intersecting lines and create a model to represent them. **Instructional Strategies** • **Interdisciplinary Connections** In chemistry, students can study refractive indices of different substances, including water and oil.

	• **Technology Integration** Students can use geogebra to create different types of intersecting lines with specific angle measures. • **Media Literacy Integration** Students can use quizlet to create study guides. • **Global Perspectives** Students can research Elizabeth Getzoff's deduction for the locations of atoms and the use of a three-dimensional coordinate system.

UNIT 5 – PARALLEL LINES AND RELATED FIGURES

Anchor Standard: G-CO: Congruence Students will be able to formally prove statements involving quadrilaterals.	
Big Ideas: *Course Objectives/Content Statement(s)* • Students will prove geometric theorems and statements involving quadrilaterals using two-column and paragraph proofs.	
Essential Questions *What provocative questions will foster inquiry, understanding, and transfer of learning?*	**Enduring Understandings** *What will students understand about the big ideas?*
How can two lines be proved parallel?	**Students will understand that…** Two lines are parallel if any of the following are true:

	a. Alternate interior angles are congruent. b. Alternate exterior angles are congruent. c. Corresponding angles are congruent. d. Consecutive interior angles are supplementary. e. Consecutive exterior angles are supplementary. f. If two lines in the same plane are perpendicular to the same line, then the lines are parallel.
What are the special quadrilaterals and how are they defined?	The special quadrilaterals are parallelograms, rectangles, squares, rhombi, kites, trapezoids, and isosceles trapezoids. They are defined by the properties of their sides, angles, and sometimes diagonals.
Areas of Focus: Proficiencies (Progress Indicators)	**Examples, Outcomes, Assessments**
Students will: G-CO-11: Prove theorems about parallelograms. *Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals.*	**Instructional Focus:** 1. Write indirect proofs. 2. Apply the exterior angle inequality theorem. 3. Prove lines parallel in two-column proofs. 4. Apply the parallel postulate. 5. Identify pairs of angles formed by a transversal cutting parallel lines. 6. Apply six theorems about parallel lines. 7. Identify special types of quadrilaterals and their properties. 8. Prove that a quadrilateral is a rectangle, parallelogram, kite, rhombus, square, trapezoid, or isosceles trapezoid.

	Sample Assessments: EX: Using an indirect proof, show that if a quadrilateral that is not a parallelogram has a diagonal drawn, that two alternate interior angles are not congruent. EX: Given quadrilateral *ABCD* where diagonals intersect at *E*, prove ΔDEC is isosceles if $\overline{AB} \cong \overline{DC}$, $\overline{AB} \perp \overline{BC}$, and $\overline{DC} \perp \overline{BC}$. EX: What is the most descriptive name for the quadrilateral with vertices (0, -6), (-4, 2), (4, 6), and (8, -2). Provide justification. **Projects/Post Assessment** Students can create their own complex diagram of parallel lines and transversals, label the angles, and challenge classmates to find the angle measures. **Instructional Strategies** • **Interdisciplinary Connections** Students can create a tangram puzzle that contains only quadrilaterals. • **Technology Integration** Students can create a Geometer's Sketchpad document that has constructions of the various quadrilaterals. This can begin the conversation about "construct vs. draw". • **Media Literacy Integration**

	Students can create a table to keep track of the properties of special parallelograms. • **Global Perspectives** Students can research the four color theorem for drawing maps.

UNIT 6 – LINES AND PLANES IN SPACE

Anchor Standard: G-MG: Modeling with Geometry Students will use static and dynamic environments with experimental and modeling tools that allow them to investigate geometric phenomena in three dimensions.	
Big Ideas: *Course Objectives/Content Statement(s)* • Students will apply geometric concepts in modeling situations with three dimensions.	
Essential Questions *What provocative questions will foster inquiry, understanding, and transfer of learning?*	**Enduring Understandings** *What will students understand about the big ideas?*
How is a plane determined? How can a line be proved perpendicular to plane? How are skew lines determined?	**Students will understand that…** A plane is determined by three noncollinear points, a line and point not on the line, two intersecting lines, or two parallel lines. If a line is perpendicular to two distinct lines that lie in a plane and that pass through its foot, then it is perpendicular to the plane. Skew lines are lines that are non-parallel and non-coplanar.
Areas of Focus: Proficiencies (Progress Indicators)	**Examples, Outcomes, Assessments**
Students will:	**Instructional Focus:**

G-MG-1: Use geometric shapes, their measures, and their properties to describe objects. G-MG-3: Apply geometric methods to solve design problems.	1. Understand basic concepts relating to planes. 2. Identify four methods of determining a plane. 3. Apply two postulates concerning lines and planes. 4. Recognize when a line is perpendicular to a plane. 5. Apply the basic theorem concerning the perpendicularity of a line and a plane. 6. Recognize lines parallel to planes, parallel planes, and skew lines. 7. Use properties relating parallel lines and planes. **Sample Assessments:** EX: A line is drawn perpendicular to the plane of a square at the point of intersection of the square's diagonals. Prove that any point on the perpendicular is equidistant from the vertices of the square. EX: Plane *m* is parallel to plane *n* and plane *p* is parallel to plane *n*. Plane *m* contains points *A* and *B*, plane *n* contains point *E* and plane *p* contains *C* and *D*. If *E* is between *A* and *D* and $\overline{AD}$ bisects $\overline{BC}$, prove $\overline{BC}$ bisects $\overline{AD}$. EX: From the top of a flagpole 48ft in height, two 60-foot ropes reach two points on the ground, each of which is 36 feet from the pole. If the ground is level, is the pole perpendicular to the ground? **Projects/Post Assessment** Students can identify where in the real world they see the different types of intersecting planes and create models of them.

	Instructional Strategies • **Interdisciplinary Connections** Students can investigate how lines and planes are used in construction by talking to the wood technology teacher. • **Technology Integration** Students can use geogebra to create digital models of three-dimensional spaces. • **Media Literacy Integration** Students can research Georges-Louis Leclerc's findings with probability and geometry. • **Global Perspectives** Students can explore architecture and understand the need that architects have for the geometry in this unit.

UNIT 7 – POLYGONS AND SIMILARITY

Anchor Standard: G-SRT Similarity, Right Triangles, and Trigonometry Students should be able to recognize that polygons have important characteristics tied to their number of sides. Students should also understand that the criterion for triangle similarity includes having two pairs of congruent, corresponding angles.	
Big Ideas: *Course Objectives/Content Statement(s)* • Students will understand and apply theorems about polygons. • Students will recognize characteristics of regular figures. • Students will prove pairs of triangles similar. • Students will prove theorems and other true statements involving similarity.	
Essential Questions	**Enduring Understandings** *What will students understand about the big ideas?*

What provocative questions will foster inquiry, understanding, and transfer of learning?	
What are regular figures? What are similar figures? How can two polygons be proved similar? What are the types of special quadrilateral? If two triangles are similar, how can additional statements (similarity, congruency, and equality) be developed?	**Students will understand that…** Regular figures are shapes where the angles and the sides are all congruent. Similar figures are figures that have the same shape but not necessarily the same size. Similar polygons are polygons in which the ratios of the measures of corresponding sides are equal and corresponding angles are congruent. For triangles, it is common to use the following theorems: AA~, SSS~, and SAS~. The following theorems can be used if two triangles are similar: a. The ratio of the perimeters of two similar polygons equals the ratio of any pair of corresponding sides. b. If a line is parallel to one side of a triangle and intersects the other two sides, it divides those two sides proportionally.
Areas of Focus: Proficiencies (Progress Indicators)	**Examples, Outcomes, Assessments**
Students will: G-SRT-2: Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides	**Instructional Focus:** 1. Use ratios and proportions to solve problems. 2. Apply the product and ratio theorems. 3. Apply the no choice theorem. 4. Calculate geometric means. 5. Identify characteristics of similar figures to prove triangles similar. 6. Apply formulas related to polygons. 7. Identify characteristics of regular figures.

G-SRT-3: Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar.

G-SRT-4: Prove theorems about triangles. *Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity*

G-SRT-5: Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.

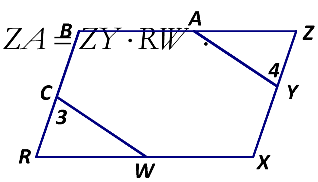
G-GPE-4: Use coordinates to prove simple geometric theorems algebraically. *For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point (1, √3) lies on the circle centered at the origin and containing the point (0, 2).*

8. Use the concept of similarity to establish the congruence of angles and the proportionality of segments.
9. Apply theorems to establish proportionality.

Sample Assessments:

EX: Triangle *OKP* has points *M* and *R* on sides $\overline{OK}$ and $\overline{KP}$, respectively. Given $\overleftrightarrow{MR} \parallel \overleftrightarrow{OP}$, *MK* = 6, *OM* = 3, *KR* = 8, *MR* = 10, find *RP* and *OP*.

EX: Given $BRXZ$ is a parallelogram and $\angle 3 \cong \angle 4$. Prove $RC \cdot ZA = ZY \cdot RW$.



EX: Triangle *AEC* has points *B*, *D*, and *F* on sides $\overline{AC}$, $\overline{CE}$, and $\overline{EA}$, respectively. Given *ABDF* is a parallelogram, prove $\Delta CBD \sim \Delta DFE$.

EX: Prove that diagonals of a trapezoid divide each other proportionally.

Projects/Post Assessment

Students will use geogebra to discover how the more sides a regular polygon has, the closer it becomes to a circle, taking on most of the characteristics of a circle, and approximating pi in the process.

Instructional Strategies

- **Interdisciplinary Connections**

	Students can write a book report and analysis based on the sketchbook of Villard de Honnecourt. • **Technology Integration** Students can use the dilation and rotation tools in geogebra to create dynamic sketches that show the maintenance of ratios in similar triangles. • **Media Literacy Integration** Students can research the many online applets that can been created and critique the usefulness of the applets based on the demands of an advanced course in geometry. • **Global Perspectives** Students can research the Vegreville Egg and how a computer scientist used technology and polygons to create the sculpture.

UNIT 8 – TRANSFORMATIONS

Anchor Standard: G-C0: Congruence, G-SRT: Similarity, right triangles, and trigonometry. Students will explore transformations on the coordinate plane.	
Big Ideas: *Course Objectives/Content Statement(s)* • Students should understand congruence in terms of rigid transformations, • Students should understand similarity in terms of non-rigid transformations.	
Essential Questions *What provocative questions will foster inquiry, understanding, and transfer of learning?*	**Enduring Understandings** *What will students understand about the big ideas?*

What is a transformation? How is a transformation done? What are the types of transformations?	**Students will understand that…** In geometry a transformation refers to the change in location and/or size of a figure in the coordinate plane through a series of movements. A transformation takes a pre-image and manipulates it in one of four ways to create an image. There are four basic types of transformations: reflection, rotation, translation, and dilation. All of the types except dilation maintain the shape and size of the original figure, making them congruent. In a dilation, the shape of the figure is maintained by the size is changed creating a similar figure.
Areas of Focus: Proficiencies (Progress Indicators)	**Examples, Outcomes, Assessments**
Students will: G-CO-2: Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch). G-CO-3: Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself. G-CO-4: Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments. G-CO-5: Given a geometric figure and a rotation, reflection, or translation, draw the	**Instructional Focus:** 1. Understand basic concepts relating to transformations. 2. Identify four methods of transforming a figure. 3. Recognize transformations as functions that take an input (pre-image) and produce an output (image). 4. Apply the different types of transformations to create an image from pre-images. 5. Recognize how to map an image back to its pre-image and vice versa given the transformation. **Sample Assessments:** EX. Take the triangle with coordinates (2,3), (5,-7) and (-3, 4) and rotate it 90

transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another. G-CO-6: Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent. G-SRT-1: Verify experimentally the properties of dilations given by a center and a scale factor: a. A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged. b. The dilation of a line segment is longer or shorter in the ratio given by the scale factor	degrees about the x-axis. What are the new coordinates? EX. An image at points (4,2) and (3,4) was created by a dilation with a scale factor of 2 from point (5,3). Find the coordinates of the pre-image. EX. Given the rectangle with points (0,0), (5,0), (5,3) and (0,3); describe the transformations that would map the figure onto itself. **Projects/Post Assessment** Students will complete a project where they create an intricate pre-image on the coordinate plane and then translate it in each of the four ways. **Instructional Strategies** • **Interdisciplinary Connections** Students can explore how artists use dilation to create art work on a grand scale. • **Technology Integration** Students will use desmos to explore each type of transformation and help discover the transformation rules. • **Media Literacy Integration** Students can use geogebra to create an image and practice transforming it in different ways. • **Global Perspectives** Students can explore how professionals such as architects and constructions workers use dilations and scale factors to make blue prints and create homes from the blue prints.

UNIT 9 – PYTHAGOREAN THEOREM

Anchor Standard: G-SRT: Similarity, Right Triangles, and Trigonometry Students should be able to discover and use properties of right triangles, including basic right triangle trigonometry.	
Big Ideas: *Course Objectives/Content Statement(s)* ● Students will understand, apply, and prove the Pythagorean Theorem. ● Students will apply special right triangle rules to solve problems. ● Students will define trigonometric ratios and solve problems involving right triangles. ● Students will apply trigonometry to general triangles	
Essential Questions *What provocative questions will foster inquiry, understanding, and transfer of learning?*	**Enduring Understandings** *What will students understand about the big ideas?*
How is the Pythagorean Theorem used? How are special right triangles used to solve problems?	**Students will understand that…** In a right triangle, the sum of the squares of the leg is equal to the square of the hypotenuse. This equation can be used to solve for missing side lengths of a right triangle. If the acute angles of a right triangle measure 30˚ and 60˚, the sides have ratio $1:\sqrt{3}:2$. If the acute angles of a right triangle measure 45˚, thus forming an isosceles triangle, the sides have ratio $1:1:\sqrt{2}$. Missing side lengths can be found using proportions with these ratios. The sine ratio is defined as the measure of the side opposite the reference angle to the

How can basic right triangle trigonometry be used to solve problems?	measure of the hypotenuse. The cosine ratio is defined as the measure of the side adjacent to the reference angle to the measure of the hypotenuse. The tangent ratio is defined as the measure of the opposite side to the measure of the adjacent side. To solve problems, students can use proportions and inverse trigonometric functions.
Areas of Focus: Proficiencies (Progress Indicators)	**Examples, Outcomes, Assessments**
Students will: G-SRT-6: Understand that by similarity, side, ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles. G-SRT-7: Explain and use the relationship between the sine and cosine of complementary angles G-SRT-8: Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems. G-SRT-9: (+) Derive the formula A = 1/2 ab sin(C) for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side. G-SRT-10: (+) Prove the Law of Sines and Cosines and use them to solve problems. G-SRT-11: (+) Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles.	**Instructional Focus:** 1. Simplify radical expressions and solve quadratic equations. 2. Identify the relationships between the parts of a right triangle when an altitude is drawn to the hypotenuse. 3. Use the Pythagorean Theorem and its converse. 4. Use the distance formula to compute lengths of segments in the coordinate plane. 5. Recognize Pythagorean triples to apply the principle of the reduced triangle. 6. Solve problems using special right triangles and ratios. 7. Understand and use the three basic trigonometric ratios. **Sample Assessments:** EX: The long leg of a 30-60-90 triangle measures 4cm. Find the exact area of the triangle. EX: A surveyor is 100m away from the base of a building. If the angle of elevation from the surveyor to the top of the building is 32˚, find the height of the building to the nearest meter. EX: Sally leaves the library and heads home, walking 15 blocks east. She then walks 10 blocks north followed by 3 blocks west to stop

	at the market. Finally, Sally walks 12.5 blocks north to her apartment. EX: A boat is tied to a pier by a 25' rope. The pier is 15' above the boat. If 8' of the rope is pulled in, how many feet will the boat move forward? **Projects/Post Assessment** Students will work in groups to prove the law of sines and law of cosines using right triangle trig and altitudes drawn on the hypotenuse. **Instructional Strategies** • **Interdisciplinary Connections** Students can solve physics problems involving projectile motion at different angles using $x(t) = vt\cos\theta$ and $y(t) = vt\sin\theta - \frac{1}{2}gt^2$. • **Technology Integration** Students can create an excel document or graphing calculator algorithm to organize and analyze data collected on self-constructed special right triangles to discover ratios. • **Media Literacy Integration** After research, students can create presentations on the many proofs of the Pythagorean theorem, including visual representations and manipulatives. • **Global Perspectives** Students can discuss a timeline of discoveries with right triangles and juxtapose it with a

	timeline of historical events to discover possible influences.

UNIT 10 – CIRCLES

Anchor Standard: G-C Circles Students should be able to recognize parts of circles, solve problems involving circles, write equations of circles, and write proofs involving circles.	
Big Ideas: *Course Objectives/Content Statement(s)* ● Students will identify parts of circles. ● Students will understand and apply theorems about circles. ● Students will find arc lengths and areas of sectors of circles. ● Students will use equations of circles on the coordinate plane.	
Essential Questions *What provocative questions will foster inquiry, understanding, and transfer of learning?*	**Enduring Understandings** *What will students understand about the big ideas?*
	Students will understand that…
How are parts of a circle related to one another?	Every circle has a center. By joining the center and any point on the circle, a radius is drawn. By joining any two points on a circle, a chord is drawn. The longest chord (the diameter) can be constructed by passing through the center of the circle. Chords, diameters, and radii create arcs, central angles, and inscribed angles. Radii and diameters are also useful in computing and discussing area, circumference, arc length and other related measures.
What relationships are found in circles when tangents, secants, tangent segments, and	After constructing special segments to and in circles, many theorems (such as the power theorems, those including perpendicular

secant segments, and chords are draw to and in a circle? How does inscribing or circumscribing polygons create new properties?	bisectors and secant-secant angles) can be defined, discussed, and proven. If a quadrilateral is inscribed in a circle, its opposite angles are supplementary. If a parallelogram is inscribed in a circle, it must be a rectangle. If the nine-point circle is circumscribed in a triangle, the triangle is equilateral.
Areas of Focus: Proficiencies (Progress Indicators)	**Examples, Outcomes, Assessments**
Students will: G-C-1: Prove all circles are similar. G-C-2: Identify and describe relationships among inscribed angles, radii, and chords. G-C-3: Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle. G-C-4: Construct a tangent line from a point outside a given circle to the circle. G-C-5: Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality, derive the formula for the area of a sector G-GPE-1: Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.	**Instructional Focus:** 1. Identify and apply the characteristics and relationships of circles, chords, diameters, and radii. 2. Relate congruent arcs, chords, and central angles. 3. Identify secant and tangent lines and segments. 4. Determine the measures of central, inscribed, tangent-chord, chord-chord, secant-secant, secant-tangent, and tangent-tangent angles. 5. Recognize inscribed and circumscribed polygons and their properties. 6. Apply the three power theorems. 7. Determine circle circumference and arc length. 8. Write and identify equations of circles on the coordinate plane. **Sample Assessments:** EX: Find the exact circumference of a circle in which an 80cm chord is 9cm from the center. EX: $\overline{TP}$ is a tangent segment of circle *O* and measures 15cm. $\overline{OP}$ intersects the circle at point *Q* and $PQ = 5$cm. Find the radius of the circle.

EX: Two secant segments, $\overline{PB}$ and $\overline{PA}$ intersect a circle at points C and D respectively. Given $m\overset{\frown}{AD} + m\overset{\frown}{BC} = 200°$ and $m\angle P = 30°$, find $m\overset{\frown}{AB}$ and $m\overset{\frown}{CD}$.

EX: A parallelogram with sides 4 and 7.5 is inscribed in a circle. Find the radius of the circle.

Projects/Post Assessment
Students will research other conic sections and their equations and compare/contrast them to the equation of the circle.

Instructional Strategies

- **Interdisciplinary Connections**

Students can apply common internal and external tangents to explain how a lunar eclipse occurs.

- **Technology Integration**

Students can create a nine-point circle using Geometer's sketchpad using previously learned material on special segments in triangles.

- **Media Literacy Integration**

Students can research art forms that include circles, for example, Kandinsky's *Concentric Circles* as inspiration for creating their own art or for learning new vocabulary words.

- **Global Perspectives**

Students can research artists who use circles in their work.

UNIT 11 – AREA

<table>
<tr><td colspan="2" align="center">Anchor Standard: 7.G Geometry

Students should be able to compute the area of polygons.</td></tr>
<tr><td colspan="2">Big Ideas: Course Objectives/Content Statement(s)
● Students will find the area of regular polygons, special quadrilaterals.
● Students will solve problems involving area.</td></tr>
<tr><td align="center">Essential Questions
What provocative questions will foster inquiry, understanding, and transfer of learning?</td><td align="center">Enduring Understandings
What will students understand about the big ideas?</td></tr>
<tr><td>How can area be computed?

How can ratios be used to solve area problems?</td><td>Students will understand that…

Formulas for area include

a. triangle - $A = \frac{1}{2}bh$
b. rectangles and parallelograms – $A = bh$
c. squares – $A = s^2$
d. trapezoid - $A = \frac{1}{2}\left(b_1 + b_2\right)h$
e. regular polygon - $A = \frac{1}{2}aP$

If two polygons are similar, the ratio of the area of one polygon to the area of the second polygon is equal to the square of the similarity ratio. Problems can be solved using this idea and proportions.</td></tr>
<tr><td align="center">Areas of Focus: Proficiencies
(Progress Indicators)</td><td align="center">Examples, Outcomes, Assessments</td></tr>
<tr><td>Students will:</td><td>Instructional Focus
1. Use and understand the basic properties of area.</td></tr>
</table>

G-GPE-7: Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula. 7.G.6: Solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms. G-MG-2:Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot).	2. Find the areas of rectangles, squares, parallelograms, triangles, trapezoids, and kites. 3. Find the areas of equilateral triangles and other regular polygons. 4. Find the areas of circles, sectors, and segments. 5. Find and use ratios of areas to solve problems. 6. Use formulas to solve problems involving area. **Sample Assessments:** EX: One angle of a parallelogram measures 60˚. If one sides measures 8cm and one sides measures 12cm, find the area of the parallelogram. EX: The area of a sector of a circle is 14π. If the radius of the circle is 2, find the central angle that forms the sector. EX: Two similar equilateral triangles have areas $49cm^2$ and $64cm^2$. Find the ratio of their perimeters. EX: Four congruent circles fit snuggly inside one larger circle. If each small circle has a radius of 3cm, find the area of the space between the four small circles and the single large circle. **Projects/Post Assessment** Students will create a complex figure made of many different polygons and calculate the area of the figure. **Instructional Strategies** • **Interdisciplinary Connections**

	Students can create a historical timeline that includes the first discussions of the concept of area and the establishment of area formulas. • **Technology Integration** Students can construct similar figures to test or discover the area ratio property using geogebra. • **Media Literacy Integration** Students can collect research on the proofs of area formulas. • **Global Perspectives** Students can research the mathematicians who proved the area formulas.

UNIT 12 – VOLUME

Anchor Standard: G-GMD: Geometric Measurement and Dimension Students should be able to define variables in surface area and volume formulas. Students should also be able to apply these formulas.
Big Ideas: *Course Objectives/Content Statement(s)* ● Students will explain volume formulas and use them to solve problems. ● Students will visualize relationships between two-dimensional and three-dimensional objects.

Essential Questions *What provocative questions will foster inquiry, understanding, and transfer of learning?*	**Enduring Understandings** *What will students understand about the big ideas?*

How are formulas for surface area of three-dimensional figures derived? How are formulas for volume of three-dimensional figures derived? How can the area of a prism's or cylinder's cross section be used to find a solid's volume? How is a cross section of a pyramid or cone used to solve problems?	**Students will understand that…** The surface area of a three-dimensional figure is the sum of the areas of each surface that forms the exterior of the figure. The volume of a prism or cylinder is the product of the area of the base of the figure and the height of the figure. The volume of a pyramid or cone is one-third this same product. The volume of a prism or cylinder can also be defined as the product of the area of a cross section of the figure and the height of the figure. This is true since a cross section of a prism or cylinder is congruent to the figure's base. In a pyramid or cone, the ratio of the area of a cross section to the area of the base equals the square of the ratio of the figures' respective distances from the vertex.
Areas of Focus: Proficiencies (Progress Indicators)	**Examples, Outcomes, Assessments**
Students will: G-GMD-1: Given an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. *Use dissection arguments, Cavalieri's principle, and informal limit arguments* G-GMD-2: Given an informal argument using Cavalieri's principle for the formulas for the volume of a sphere and other solid figures.	**Instructional Focus:** 1. Find the surface area of prisms, pyramids, circular solids. 2. Find the volumes of prisms, cylinders, pyramids, cones, and spheres. 3. Use the area of a prism's or a cylinder's cross section to find the solid's volume. 4. Solve problems involving cross sections of pyramids and cones. **Sample Assessments:**

G-GMD-3: Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems. G-GMD-4: Identify the shapes of two-dimensional cross-sections of three-dimension objects, and identify three-dimensional objects generates by rotations of two-dimensional objects.	EX: Find the surface area of a cube with a volume of 216cm^3. EX: Find the volume of a cylindrical glass if its height is 15cm and a 17cm straw just fits inside it. EX: A right triangle with one acute angle measuring 30˚ has hypotenuse 8cm. If the triangle is rotated about the long leg, find the volume of the resulting figure. EX: A frustum of a cone has base radii 9cm and 12cm. If the slant height is 6cm, find the volume of the solid. **Projects/Post Assessment** Students will construct a 3D model of box that has been scaled down by a portion of the larger size. Students must be precise in their measurements. They should calculate the volume of the new figure based on the portion of the whole figure it is based on. **Instructional Strategies** • **Interdisciplinary Connections** Applications to chemistry include finding the volume of a liquid based on the dimensions of a beaker. • **Technology Integration** Students can use the website http://www.mathsnet.net/geometry/solid/index.html to explore and manipulate three-dimensional solids. • **Media Literacy Integration** Using the website listed above, students can explore the “Cube Houses” applet to create

	their own cube house, a silhouette, and four-way view that models a structure of their choice. • **Global Perspectives** Applications to the real world can include the following: EX: A carpenter drills a hole (1-inch in diameter) into a 2x4 that is three-quarters of a foot long. Find the volume of the wood after the hole is drilled.

UNIT 13 – ALGEBRA EXTENSION

<table>
<tr><td colspan="2">Anchor Standard: G-GPE – Expressing Geometric Properties with Equations

Students will explore Geometry in the coordinate plane and enrich their understanding of the connection between geometry and algebra.</td></tr>
<tr><td colspan="2">Big Ideas: Course Objectives/Content Statement(s)
• Students will apply the principles of coordinate geometry in a variety of situations.
• Students will graph in three dimensions.
• Students understand how conic sections are created.</td></tr>
<tr><td>Essential Questions
What provocative questions will foster inquiry, understanding, and transfer of learning?</td><td>Enduring Understandings
What will students understand about the big ideas?</td></tr>
<tr><td>What is coordinate geometry?</td><td>Students will understand that…

Coordinate geometry is a branch of geometry where the position of points on a plane is defined with ordered pairs. The ordered pairs form relations, and for the purposes of geometry they will often form familiar shapes.</td></tr>
</table>

How do you graph in three dimensions? What are conic sections?	Graphing in three dimensions adds an additional axis to the traditional x, y system. We call the third dimension the z-axis. Coordinates will have three parts; (x,y,z). Students will draw a three dimensional coordinate space and then follow the axis to arrive at the point. A conic section is a figure formed by the intersection of a plane and a cone. Depending on the angle of the plane with respect to the cone, a conic section may be a circle, ellipse, parabola, or hyperbola.
Areas of Focus: Proficiencies (Progress Indicators)	**Examples, Outcomes, Assessments**
Students will: G-GPE-2: Derive the equation of a parabola given a focus and directrix G-GPE-3: (+) Derive the equations of ellipses and hyperbolas given the foci, using the fact that the sum or difference of distances from the foci is constant. G-GPE-5: Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).	**Instructional Focus:** 1. Draw lines and circles that represent solutions of equations. 2. Identify parallel and perpendicular lines on the coordinate plane. 3. Graph coordinates in three dimensions. 4. Write equations that correspond to lines, circles, and parabolas. 5. Identify the type of conic section from a graph or an equation. 6. Write the equation of a conic section given the focus and directrix. **Sample Assessments:** EX. Write the equation of each circle. A) center (2, -3), radius 4. B) End points of the diameter are (5, 6) and (9,10). EX. Graph a parabola whose focus is (2, -5) and directrix is y=3. EX. Graph the equations x + 3y = 10 and y = -1x +3. Are the lines parallel,

	perpendicular, or neither? **Projects/Post Assessment** Students will create graphs of each of the conic sections and show how each equation is derived. **Instructional Strategies** • **Interdisciplinary Connections** Students will research how coordinate geometry is used in physics, engineering, and aviation. • **Technology Integration** Students will use desmos to explore the different types of conic sections and discover the features of each graph. • **Media Literacy Integration** Students can use geogebra to accurately graph coordinate figures in three dimensions. • **Global Perspectives** Students will research the mathematician Bhaskara from medieval India and learn about his problem called ‘The Serpent and the Peacock’.

Supports for English Language Learners		
Sensory Supports	**Graphic Supports**	**Interactive Supports**
Real life objects	Charts	In pairs or partners
Manipulatives	Graphic Organizers	In triands or small groups
Pictures	Tables	In a whole group
Illustrations, diagrams & drawings	Graphs	Using cooperative group
Magazines & Newspapers	Timelines	Structures
Physical activities	Number lines	With the Internet / Software
Videos & Film		In the home language
Broadcasts		With mentors
Models & Figures		
Intervention Strategies		
Accommodations	Interventions	Modifications
Allow for verbal responses	Multi-sensory techniques	Modified tasks/expectations
Repeat/confirm directions	Increase task structure (e.g. directions, checks for understanding, feedback	Differentiated materials
Permit response provided via computer or electronic device	Increase opportunities to engage in active academic responding	Individualized assessment tools based on student need
Audio Books	Utilize pre reading strategies and activities previews, anticipatory guides, and semantic mapping	Modified assessment grading

Career-Ready Practices:
CRP1: Act as a responsible and contributing citizen and employee.
CRP2: Apply appropriate academic and technical skills.
CRP3: Attend to personal health and financial well-being.
CRP4: Communicate clearly and effectively and with reason.
CRP5: Consider the environmental, social and economic impacts of decisions.
CRP6: Demonstrate creativity and innovation.
CRP7: Employ valid and reliable research strategies.
CRP8: Utilize critical thinking to make sense of problems and persevere in solving them.
CRP9: Model integrity, ethical leadership and effective management.
CRP10: Plan education and career paths aligned to personal goals.
CRP11: Use technology to enhance productivity.
CRP12: Work productively in teams while using cultural global competence.

References

McDougal, Littell: Geometry, For Enjoyment and Challenge

New Jersey Standards for Mathematics (2016),
https://www.state.nj.us/education/cccs/2016/math/standards.pdf

Summit Public Schools

Summit, New Jersey

Curricular Addendum

Career-Ready Practices	Interdisciplinary Connections
CRP1: Act as a responsible and contributing citizen and employee. **CRP2**: Apply appropriate academic and technical skills. **CRP3**: Attend to personal health and financial well-being. **CRP4**: Communicate clearly and effectively and with reason. **CRP5**: Consider the environmental, social and economic impacts of decisions. **CRP6**: Demonstrate creativity and innovation. **CRP7**: Employ valid and reliable research strategies. **CRP8**: Utilize critical thinking to make sense of problems and persevere in solving them. **CRP9**: Model integrity, ethical leadership and effective management. **CRP10**: Plan education and career paths aligned to personal goals. **CRP11**:. Use technology to enhance productivity. **CRP12**: Work productively in teams while using cultural global competence.	• Close Reading of works of art, music lyrics, videos, and advertisements • Use Standards for Mathematical Practice and Cross-Cutting Concepts in science to support debate/inquiry across thinking processes **Technology Integration** Ongoing: • Listen to books on CDs, Playaways, videos, or podcasts if available. • Use document camera or overhead projector for shared reading of texts. Other: • Use Microsoft Word, Inspiration, or SmartBoard Notebook software to write the words from their word sorts. • Use available technology to create concept maps of unit learning.

Instructional Strategies:
Supports for English Language Learners:

Sensory Supports	Graphic Supports	Interactive Supports
Real-life objects (realia)	Charts	In pairs or partners
Manipulatives	Graphic organizers	In triads or small groups
Pictures & photographs	Tables	In a whole group
Illustrations, diagrams, & drawings	Graphs	Using cooperative group structures
Magazines & newspapers	Timelines	With the Internet (websites) or software programs
Physical activities	Number lines	In the home language
Videos & films		With mentors
Broadcasts		
Models & figures		

from https://wida.wisc.edu

Media Literacy Integration

- Use multiple forms of print media (including books, illustrations/photographs/artwork, video clips, commercials, podcasts, audiobooks, Playaways, newspapers, magazines) to practice reading and comprehension skills.

Global Perspectives

- [The Global Learning Resource Library](#)

Differentiation Strategies:

Accommodations	Interventions	Modifications
Allow for verbal responses	Multi-sensory techniques	Modified tasks/ expectations
Repeat/confirm directions	Increase task structure (e.g., directions, checks for understanding, feedback)	Differentiated materials
Permit response provided via computer or electronic device	Increase opportunities to engage in active academic responding (e.g., writing, reading aloud, answering questions in class)	Individualized assessment tools based on student need
Audio Books	Utilize prereading strategies and activities: previews, anticipatory guides, and semantic mapping	Modified assessment grading