Summit Public Schools

Summit, New Jersey

Grade Levels 11th - 12th / Content Area: Mathematics

Length of Course: Full Academic Year

Pre-Calculus Honors

Submitted by:

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Curriculum

Course Description:

Pre-Calculus Honors is a full-year course offered as part of the mathematics curriculum. This course begins with a review of important parent functions, focusing on their properties and applications. An in-depth study of trigonometry will follow, expanding upon previously learned material. Students will solve problems that arise in a variety of fields using algebraic and graphic properties of trigonometry, often requiring the application of important identities. The course will continue with the study of vectors, parametric and polar equations, conic sections, sequences, series, the Binomial Theorem, and an introduction to limits and calculus. Throughout the course, students will explore the application and relevance of these topics in a variety of real-world scenarios. Students will be expected to demonstrate both facility with the algebraic skills presented and the ability to discuss the complex topics being explored. Students will be expected to use a variety of technological tools in solving the complex problems that they will encounter in this course. Skills in using the TI-83Plus graphing calculator and a variety of computer programs, such as Desmos, will be taught in parallel with the material.

(Corresponding textbook sections are included in parentheses)

Unit 1 – Functions and Their Graphs

Topic	Time Frame
Algebraic Functions (1.1, 1.2)	4
Function Transformations (1.3, 1.6)	4
Function Operations (1.4, 1.5)	4
Review	1
Assessments	2
Total	15

Topic	Time Frame
Functions, Graphically (7.1, 7.2)	3
Functions, Numerically (7.3)	4
Logarithmic and Exponential Functions (7.4, 7.5)	4
Logistic Functions (7.6)	4
Review	1
Assessments	2
Total	18

Unit 2 – Review of Trigonometry

Periodic Functions (2.1, 2.2, 2.3)	2
Trigonometric Function Values (2.4)	2
Inverse Trig Functions (2.5)	2
Graphs of the Six Trigonometric Functions (3.1, 3.2, 3.3)	3
Review	1
Assessments	2
Total	12

Unit 3 – Trigonometric Graphs and Identities

Topic	Time Frame
Radian Angle Measurement (3.4)	1
Circular Functions (3.5, 3.6)	2
Modeling with Sinusoidal Functions (3.7)	2
Pythagorean and Reciprocal Identities (4.1, 4.2, 4.3)	1
Inverse Trig Functions and Graphs (4.4, 4.6)	2
Parametric Functions (4.5)	1
Review	1
Assessments	2
Total	12

Unit 4 – Applications of Trigonometry

Composite Arguments and Linear Combinations (5.2, 5.3)	3
Composition of Ordinates and Harmonic Analysis (5.4)	3
Sum, Product, Double, and Half Angle Identities (5.5, 5.6)	3
Review	1
Assessments	2
Total	12

Law of Cosines (6.2)	1
Law of Sines (6.4, 6.5)	2
Area of Triangles (6.3)	2
Real-World Triangle Problems (6.7)	2
Review	1
Assessments	2
Total	10

Unit 5 – Vectors

Topic	**Time Frame**
Two-Dimensional Vectors (6.6, 10.1, 10.2)	3
Vectors in Space (10.3)	2
Scalar Products and Vector Projections (10.4)	4
Planes in Space (10.5)	3
Vector Product of Two Vectors (10.6)	3
Direction Angles and Direction Cosines (10.7)	2
Review	1
Assessments	2
TOTAL	20

Unit 6 – Conic Sections

Equations and Graphs of the Conic Sections (12.1, 12.2)	3
Quadratic Surfaces and Inscribed Figures (12.3)	3
Analytic Geometry of the Conic Sections (12.4)	3
Parametric Equations of Conic Sections (12.5)	3
Applications of Conic Sections (12.6)	3
Review	2
Assessments	3
TOTAL	20

Unit 7 – Polar Coordinates and Complex Numbers

Topic	Time Frame
Polar Equations (13.1, 13.2)	3
Intersections of Polar Curves (13.3)	3
Complex Numbers in Polar Form (13.4)	3
Parametric Equations for Motion (13.5) **optional, as time allows	2
Review	1
Assessments	2
TOTAL	14

Unit 8 – Discrete Math and an Introduction to Calculus

Arithmetic, Geometric and Other Sequences (14.1, 14.2)	4
Series and Partial Sums (14.3)	4
Binomial Theorem (14.3)	4
Review	1
Assessments	2
TOTAL	15

Review of Polynomial Functions, Graphs, and Zeros (15.1, 15.2, 15.3)	3
Rational Functions, Limits and Continuity (15.4)	4
Instantaneous Rate of Change, the Derivative of $f(x)$ (15.5)	4
Mathematical Induction (not in text)**optional as time allows	2
Review	1
Assessments	2
TOTAL	16

Unit 1 - Functions and Their Graphs

Anchor Standards: Interpreting Functions (F-IF), Building Functions (F-BF), Arithmetic with Polynomials (A-APR)	
Big Ideas: *Course Objectives / Content Statement(s)* • To review functions and learn to identify, categorize, describe and graph functions. • To express numerical relationships algebraically, graphically, and numerically. • To examine, with great detail, polynomial, rational, logarithmic, exponential, and logistic functions.	
Essential Questions *What provocative questions will foster inquiry, understanding, and transfer of learning?*	**Enduring Understandings** *What will students understand about the big ideas?*
What are the physical and algebraic characteristics of functions?	Students will understand that… Translations, dilations, and reflections can be applied to any parent function by changing parameters in the parent function's equation.
How can functions be represented in different ways?	Functions can be represented in algebraic, graphic, numeric, and verbal forms.
What are the important characteristics of, and similarities and differences between, the important families of functions?	Polynomial, rational, exponential, logarithmic, and logistic functions each have defining characteristics such as a domain, range, end behavior, and critical points. Functions can model real world scenarios in order to calculate, analyze, and predict outcomes

Areas of Focus: Proficiencies (Cumulative Progress Indicators)	**Examples, Outcomes, Assessments**
Students will: • (A-APR-1) Perform arithmetic operations on polynomials • (A-APR-2,3) Understand the relationship between zeros and factors of polynomials • (A-APR-4,5) Use polynomial identities to solve problems • (F-BF-3) Identify horizontal and vertical translation of functions ▪ (F-BF-3) Reflect the graph of a function in the x- or y-axis ▪ (F-BF-3) Learn how to stretch graphs ▪ (F-BF-3) Perform arithmetic combinations of functions ▪ (F-BF-1c) Compositions of Functions ▪ (F-BF-4) Determining if a function has an inverse ▪ (F-BF-4) Find the inverse of functions ▪ (F-BF-5) Understand inverse relationship between exponents and logarithms ▪ (F-IF-7A) Graph quadratic functions ▪ (F-IF-8A) Identify and write the standard form of a quadratic function ▪ (F-IF-7C) Graph of higher degree polynomials ▪ (F-IF-7C) Use the leading coefficient test to determine the end behavior of polynomial functions ▪ (F-IF-7C) Find zeros of polynomial functions ▪ (F-IF-7C) Write the polynomial function given the zeros of a function ▪ (F-IF-7) Perform long division of polynomials ▪ (F-IF-7) Apply the Remainder Theorem to evaluate a function ▪ (F-IF-7) Apply the rational zeros of a polynomial ▪ (F-IF-7) Apply the Fundamental Theorem of Algebra ▪ (F-IF-7) Determine the bounds for real zeros of polynomial functions ▪ (F-LE-2, 3, 4, 5) Evaluating exponential expressions	Instructional Focus: SWBAT: • Identify the type of function based given numerical, graphical, verbal, or algebraic data • Identify domain and range of a given function • Provide a detailed graph of a given function • Identify transformations that have been applied to a given function • Determine graphically or algebraically that a function is odd, even, or neither • Apply transformations to the graph of a given function • Identify the domain and range of composite functions and apply the use of composite functions to solve real world applications. • Identify the inverse of a given function, recognize when functions are inverses of each other, determine if a function is invertible • Use Parametric Equation to graph a function and its inverse • Given a set of data, identify the type of function based on the numerical patterns of the data • Given a set of data, write the rule for the function that models the given data • Solve complex logarithmic and exponential equations using exponential and logarithmic properties • When given a real world application, identify the type of function that will be needed to model the given scenario Sample Assessments: 1) EX: Let A={2, 3, 4, 5} and B={-3, -2, -1, 0, 1}. Which of the following sets of ordered pairs represent functions from set A to set B? a. (-2, 2), (3, 0), (4, 1) b. (3, -2), (5, 0), (2, -3) 2) EX: Let $f(x) = 10 - 3x^2$ and find the following: a. f(2) b. f(-4) c. f(x - 1) 3) EX: Find the domain and range of $g(x) = \sqrt{x - 16}$ 4) Explain to your friend how you would classify the following equations. Describe what would each graph look like?

- (F-LE- 3 & F-IF 7e) Graphing exponential functions
- (F-LE-1c, 5) Compound interest problems
- (F-LE-4) Evaluating logarithms
- (F-LE) Solving exponential equations
- (F-LE- 4) Solving logarithmic equations
- (F-LE- 3 & F-IF 7e) Graphs of logarithmic functions
- (F-LE- 4) Rewriting logarithmic expressions
- (F-LE- 4) Condensing logarithmic expressions
- (F-LE-1c, 5) Problems that deal with logarithms and exponents
- (F-IF-7) Perform synthetic division of polynomials

a. $y = 4x + 5$

b. $x^2 - y^2 = 5$

c. $y^2 = 4x + 5$

d. $x^2 + y^2 = 5$

e. $y = |4x + 5|$

f. $y = \sqrt{x} + 4$

g. $y = x^3 + 5$

h. $y = (x + 2)^2$

5) Sketch the graph of:

$$f(x) = \begin{cases} -\frac{1}{2}x - 6, & x \le -4 \\ x + 5, & x > -4 \end{cases}$$

$$f(x) = \begin{cases} -\frac{1}{2}x - 6, & x \le -4 \\ x + 5, & x > -4 \end{cases}$$

6) Determine whether $f(x) = 5 - 3x$ is even, odd, or neither.

7) For $f(x) = x^2 - 2x + 9$, find $\frac{f(4+h) - f(4)}{h}, h \neq 0.$

8) Use the graph to describe the increasing, decreasing behavior of the function: $f(x) = x^3 + 3x^2 - 10.$

9) Graph the following function using transfomrations:

$f(x) = x^3$

$g(x) = (x - 2)^3$

$h(x) = -x^3$

10) *If* $f(x) = x^2 + 2$, *find* $\frac{f(h+1) - f(1)}{h}$, $x \neq 0.$

11) Perform the given operations and simplify.

a) $\left(3x^2+2x-1\right)+\left(4x-x^2+10\right)$

b) $\left(x^2-5\right)-\left(x^2-5x+4\right)$

c) $4\left(x^2+\frac{1}{2}\right)+5$

d) $\frac{10x^2+30x}{10x}$

13) Given: $f(x)=x^2$, $g(x)=1-x$

Find:

a) (f + g) (x)

b) (f – g)(x)

c) (f * g) (x)

d. (f / g) (x)

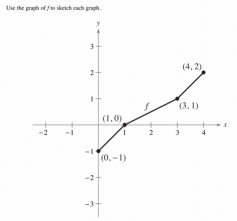
e. What is the domain of f/g?

14) Given

$f(x)=x^2$, $g(x)=\sqrt{x-6}$, *find* $(f\circ g)(-1)$.

Determine if $(f\circ g)=(g\circ f)$.

15) Given the graph shown, graph each of the following:



a) y = f(x) + 2
b) y = -f(x)
c) y= f(x-2)
d) y = f(x+3)
e) y = 2f(x)
f) y = f(-x)
g) y = f(½ x)

16) Use the function $f(x)=5^x$ to describe the transformation:

a) $g(x) = f(x) + 1$

b) $h(x) = f(x) - 3$

c) $s(x) = f(x + 2)$

d) $j(x) = f(x - 5)$

e) $c(x) = -2f(x)$

f) $t(x) = f(-x) + 4$

17) Solve the equation for x.

a. $\log_2 x = 4$
b. $\log_x 64 = 2$
c. $\log_a 1 = x$
d. $\log_a a = x$
e. $\log_a a^x = x$
f. $\log_a\left(\frac{1}{a^x}\right) = x$

18) Solve the equation for x.

a) $16 = 2^{7x-5}$
b) $3^x + 4 = 9$
c) $25^{x-2} = 5^{3x}$
d) $e^x = 60$
e) $\log(x+1) + 3 = 5$
f) $3 + 4\ln x = 31$
g) $6^{5x} = 3000$

19) The population P of a city is given by $P = 240{,}360e^{0.015t}$ where t represents years since 1990. According to this model, when will the population reach 275,000?

Sample question:

Students will analyze the path of a ball of a Major league player's hit. Students will be able to approximate how high the ball will go before it starts to fall back to the ground. In addition, students will be able to determine the length of time it will take for the ball to return to the ground. Students will be able to analyze the ball's flight graph and, based on changes to the original function, examine the impacts of those changes on the flight of the ball.

	Instructional Strategies: • Daily conversational format through which students will share and debate solutions and concepts • Use of available technology such as Desmos and the graphing calculator to explore the components of each function and compare the graph to other functions • Formative assessments throughout to assess current levels of understanding • Summative assessments will be administered at the end of each unit of study Interdisciplinary Connections Students will study functions that represent the bacteria in foods. Students will be able to find the composition of the temperature in the refrigerator and foods, the number of bacteria in the food after X amount of hours and the time when bacteria count reaches a certain amount. Exponential and logarithmic functions have many real-life applications. Some include radioactive decay, sound intensity, Newton's Law of Cooling, and the Richter scale for earthquakes Technology Integration Students will use a graphing calculator or Desmos to find maximums and minimums of functions. In addition, students will use a graphing calculator to determine when a function has a vertical stretch or shrink. Media Literacy Integration Students will research and find a report on a recent earthquake and share data, graphs, and mathematical information used to support the report. The summary will include connections to the topics discussed in class. Global Perspectives Students can approximate maximum and minimum temperatures of a particular function representing a particular city. Students will select a well-known, worldwide city and gather the monthly average temperatures. They will develop functions representing those temperatures and use the functions to compare temperatures around the world at given moments during the year. Students will research and examine the impact of earthquakes throughout the world. Each student will review the data for a given earthquake and examine how the logarithmic function can be used to model the strength (Richter scale) of that earthquake. As an

	extension, students can discuss the impact of those earthquakes on the communities where the activity occurred.

Unit 2 – A Review of Trigonometry

Anchor Standards: Trigonometric Functions (F-TF), Similarity, Right Triangles, and Trigonometry (G-SRT)	
Big Ideas: *Course Objectives / Content Statement(s)* • Students will review how angles are measured in the coordinate plane in standard position. • Students will review and apply the definitions of the six trigonometric functions. • Students will review inverse trigonometric functions and their domain restrictions.	
Essential Questions *What provocative questions will foster inquiry, understanding, and transfer of learning?*	**Enduring Understandings** *What will students understand about the big ideas?*
	Students will understand that…
What is an angle in standard position? Why might the system have been set up in this way?	Angles in standard position are measured from the positive x-axis. Positive angles rotate counter-clockwise and negative angles rotate clockwise. Such a system exploits the sign of coordinates in the Cartesian plane to define the trigonometric functions.
How can we quickly evaluate trigonometric functions?	Assuming they've memorized the 30-60-90 and 45-45-90 triangles, students can use reference angles and the sign of the coordinates in the angle's quadrant to quickly evaluate a trig function.
Why must inverse trig functions be restricted?	The trig functions are not one-to-one, and therefore are not invertible. Students must restrict the trig functions before inverting them to ensure that the inverted relations are in fact functions.
Areas of Focus: Proficiencies **(Cumulative Progress Indicators)**	**Examples, Outcomes, Assessments**
Students will:	Instructional Focus:
▪ (F-TF-2) explain how the unit circle enables the extension of trig functions to all real numbers	SWBAT:

▪ (F-TF-3) use special triangles to determine geometrically the values of trig functions ▪ (F-TF-4) use the unit circle to explain the symmetry and periodicity of trigonometric functions ▪ (F-TF-6) understand that restricting a trigonometric function to a domain on which it is always increasing or decreasing allows its inverse to be constructed ▪ (G-SRT-6) understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios • (G-SRT-7) explain and use the relationship between the sine and cosine of complementary angles ▪ (G-SRT-8) use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems	• Recognize angles in standard position and identify coterminal and reference angles • Sketch the parent sine and cosine functions and apply any necessary transformations • Given a right triangle, identify any of the six trigonometric ratios • Calculate the exact values of any of the six trigonometric functions • Find the inverse value of a given sine, cosine, or tangent expression • Use right triangle trigonometry to solve application problems Sample Assessments: 1) The initial current charge in an electrical circuit is zero. The current when 100 volts is applied to the circuit is given by $$I = 5e^{-2t}\sin t$$ where the resistance, inductance, and capacitance are 80 ohms, 20 henrys, and 0.01 farad, respectively. Approximate the current (in amperes) t=0.7 second after the voltage is applied. 2) Evaluate: a) $\csc\left(\frac{\pi}{6}\right)$ b) $\sin\left(\frac{3\pi}{2}\right)$ c) $\tan\left(\frac{5\pi}{4}\right)$ 3) Use $\sec\theta = 5,\ \tan\theta = 2\sqrt{6}$ to find the indicated trig functions. $a.\cos\theta$ $b.\cot\theta$ $c)\cot(90^\circ - \theta)$ $d)\sin\theta$ 4) Find each value of θ in degrees $(0^\circ < \theta < 90^\circ)$ and radians $\left(0 < \theta < \frac{\pi}{2}\right)$ without using a calculator. A. $\cos\theta = \frac{\sqrt{2}}{2}$ B. $\tan\theta = 1$ C. $\sec\theta = 2$ 5) TRUE OF FALSE: $A.\ \sin 170^\circ = -\sin 350^\circ$ $B.\ \cos\frac{3\pi}{7} = \cos\left(-\frac{10\pi}{7}\right)$ $C.\ -\tan 220^\circ = \tan 140^\circ$

	6) A ship leaves port at noon and has a bearing of S 27 W. If the ship sails at 20 knots (nautical miles per hour), how many nautical miles south and how many nautical miles west will the ship have traveled by 6:00 P.M.? 7) An observer in a lighthouse 350 feet above sea level observes two ships directly offshore. The angles of depression to the ships are 4 and 6.5. How far apart are the ships? Instructional Strategies: • Daily conversational format through which students will share and debate solutions and concepts • Use of available technology such as Desmos and the graphing calculator to explore the components of each function and compare the graph to other functions • Formative assessments throughout to assess current levels of understanding • Summative assessments will be administered at the end of each unit of study Interdisciplinary Connections Students will discuss how trigonometry is used in the development of astronomy, navigation, and surveying. Students will be given an irregularly shaped plot of land and apply the laws of trigonometry to evaluate the dimensions and area of that land. Technology Integration Through the use of the graphing calculator, students will examine a given function and locate all zeros, real and non-real, of the function. Students will describe the location of those zeros in relation to the x- and y-axes. The students will then apply various transformations to the function, view the corresponding graphs simultaneously, and discuss the impact it has on the zeros of those functions.

Unit 3 – Trigonometric Graphs and Identities

<table>
<tr><th colspan="2">Anchor Standards:

Trigonometric Functions (F-TF),Interpreting Functions (F-IF)</th></tr>
<tr><td colspan="2">Big Ideas: Course Objectives / Content Statement(s)

• Students will create the graphs of the six trigonometric and inverse trigonometric functions.
• Students will apply trigonometry to the system of radian angle measurement.
• Students will understand and apply basic trigonometric identities, including Pythagorean and reciprocal properties.</td></tr>
<tr><th>Essential Questions

What provocative questions will foster inquiry, understanding, and transfer of learning?</th><th>Enduring Understandings

What will students understand about the big ideas?</th></tr>
<tr><td>How can you identify the amplitude and period of a sinusoidal function from an equation or a graph?

What is the benefit of using radians to measure angles, as opposed to degrees?

How can trigonometric identities help in simplifying expressions or solving equations?</td><td>Students will understand that…

The parameters that determine amplitude and period are the same parameters that affect dilations, as described in the first unit.

Radians are a unit-less measurement, and as such do not create a conflict when trigonometric functions are combined with algebraic or other transcendental functions. Students will also appreciate the logical, geometric system that radian angles establish.

Trigonometric identities allow us to exchange trig functions or to change the argument on a trig function, both of which are helpful in creating consistency in an equation or expression.</td></tr>
<tr><th>Areas of Focus: Proficiencies

(Cumulative Progress Indicators)</th><th>Examples, Outcomes, Assessments</th></tr>
<tr><td>Students will:

▪ (F-IF-7e) graph trigonometric functions, showing period, midline, and amplitude</td><td>Instructional Focus:

SWBAT:</td></tr>
</table>

▪ (F-IF-4) interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description ▪ (F-TF-1) understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle ▪ (F-TF-5) choose trigonometric functions to model periodic phenomena with specified period, amplitude, and midline ▪ (F-TF-8) prove the trigonometric Pythagorean identities, and use them to evaluate trig functions	• Identify the amplitude, period, sinusoidal axis, and phase displacement of any of the six trigonometric functions • Provide a detailed graph of any of the six trigonometric functions in degree mode • Given a sine or cosine function, provide equivalent functions at different phase displacement values • Understand the definition of a radian measure • Recognize that angles can be measured in radians • Convert from degree to radian measure and from radian measure to degree measure • Find the exact values of all six trigonometric functions given a radian measure for the angle • Provide a detailed graph of any of the six trigonometric functions in radian mode • Find the inverse value of any of the six trigonometric functions • Apply the use of trigonometric functions while solving a real-world application • Apply all trigonometry properties while providing a proof of a trigonometric identity • Apply all trigonometry properties while solving a trigonometric equation • Provide a detailed graph of the inverse of any of the six trigonometric functions Sample Assessments: 1) Use the trig identities to transform one side into the other. $\csc\theta\tan\theta=\sec\theta$ $(\csc\theta+\cot\theta)(\csc\theta-\cot\theta)=1$ 2) Analyze and sketch the graph: $y=3\cos\left(\frac{\pi x}{2}+\frac{\pi}{2}\right)-3$ 3) Sketch at least **ONE CYCLE** of the graph of the function: $f(x)=-2\csc 4x+2$. State two points and the asymptotes in the same cycle. 4) Sketch **TWO CYCLES** of the graph of the function: $f(x)=\frac{1}{3}\cot\left(x-\frac{\pi}{4}\right)$. State two points and the asymptotes in the same cycle. 5) Evaluate: a) $\tan\left(\arctan 1\right)$ b) $\sec\left(\arcsin\frac{4}{5}\right)$

c) $\csc\left(\arctan\left(-\frac{5}{12}\right)\right)$

6) Write an algebraic expression that is equivalent to the expression: $\sec(\arctan 3x)$.

Instructional Strategies:

- Daily conversational format through which students will share and debate solutions and concepts
- Use of available technology such as Desmos and the graphing calculator to explore the components of each function and compare the graph to other functions
- Formative assessments throughout to assess current levels of understanding
- Summative assessments will be administered at the end of each unit of study

Interdisciplinary Connections

Students will discuss that trigonometry is used in applications involving vibrations, sound waves, light rays, planetary orbits, vibrating strings, pendulums and orbits of atomic particles.

Students will use an oscilloscope to gather data on sounds waves. They will recognize the connection between the mathematics and concepts of physics amongst the data points .

Technology Integration

Students will use the following website to discover the graph of the sine curve:

http://www.intmath.com/trigonometric-graphs/1-graphs-sine-cosine-amplitude.php.

Students will also use the graphing calculator and Desmos to explore the patterns and relationships between the graphs of the trigonometric functions

Media Literacy Integration

Students will research high and low tide charts that are published in a newspaper or online for a selected coastal region of their choice within the United States and share data, graphs, and mathematical information used to

	support the report. The summary will include connections to the topics discussed in class. Global Perspectives Students will research high and low tide charts for a selected coastal region of their choice outside of the United States. They will use the given data to write a cosine function that will model their data. Students will also provide a brief explanation as to why high and low tides exist.

Unit 4 – Applications of Trigonometry

Anchor Standards: Trigonometric Functions (F-TF), Similarity, Right Triangles, and Trigonometry (G-SRT)	
Big Ideas: *Course Objectives / Content Statement(s)* • Students will examine the graphical, algebraic, and numeric result of combining sinusoids. • Students will learn and apply the sum, difference, double, and half angle identities. • Students will solve oblique triangles and find their area.	
Essential Questions *What provocative questions will foster inquiry, understanding, and transfer of learning?*	**Enduring Understandings** *What will students understand about the big ideas?*
How are the periodicity, amplitude, and other defining features of sinusoids affected by adding or multiplying sinusoids? How can trigonometric functions with complex arguments be simplified?	Students will understand that… Adding or multiplying trigonometric functions with different periods create "variable sinusoids", where the critical points and amplitudes of the waves vary at regular increments. Sum and difference identities can be used to create double and half angle identities, allowing us to re-write trigonometric expressions with several terms or coefficients.
Areas of Focus: Proficiencies **(Cumulative Progress Indicators)**	**Examples, Outcomes, Assessments**

- (F-TF-9) prove the addition and subtractions formulas for sine, cosine, and tangent and use them to solve problems
- (G-SRT-9) derive the formulas for area of oblique triangles
- (G-SRT-10) prove the Law of Sines and Cosines and use them to solve problems
- (G-SRT-11) understand and apply the Law of Sines and Cosines to find unknown measurements in right and oblique triangles

Instructional Focus:

SWBAT:

- Apply the Linear Combination Property to transform the sum of two trigonometric functions with the same period onto one cosine function
- Apply the Composite Argument Properties, Odd-Even Properties, Co-Function Properties, and Double Angle Properties to transform complex trigonometric expressions into more simple expressions
- Apply the Composite Argument Properties, Odd-Even Properties, Co-Function Properties, and Double Angle Properties to solve complex trigonometric equations
- Recognize when a trigonometric equation cannot be solved algebraically and be able to provide a numerical solution
- Given the sketch of a pair of combined sinusoids, identify whether the combination is a sum or a product
- Given the sketch of a pair of combined sinusoids, write a combined function that will model the given sketch
- Provide a proof of the Law of Cosines, the Law of Sines, and the formula for the Area of an Oblique Triangle
- Understand under which conditions each law or formula can be applied
- Given a triangle, apply the appropriate Law or formula to solve the problem
- Identify when an Ambiguous Case triangle has been presented
- Given an Ambiguous Case triangle, identify the number of solutions and find the necessary values
- Apply the use of the Laws and formulas when solving a real-world problem

Sample Assessments:

1. Give a numerical example of a pair of co-functions. Explain why they are co-functions.

2. *sin x* is considered an *odd function*. Explain why.

3. Show numerically that the statement *sin (A + B) = sin A + sin B* is false

4. Express the equation below as a linear combination of cosine and sine.

$$y = 12\cos(x - 4)$$

5. Express the equation below as a single cosine function.

$$y = -3\cos\theta + 2\sin\theta$$

Solve each equation algebraically. When necessary, round all answers to the nearest 100th.

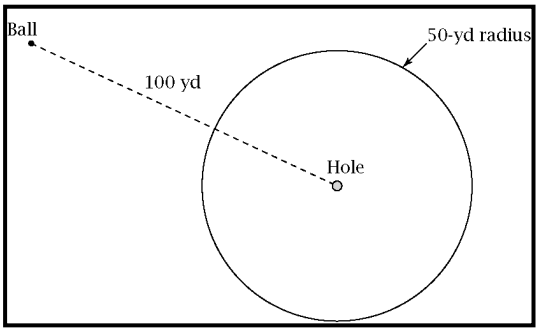
6. $-2\cos\theta + 5\sin\theta = 2$ for $\theta \in [0, 360^\circ]$

7. $6\sin x \cos x = 2.46$ for $x \in [0, 2\pi]$

8. A coast guard station at point X is located 54 miles north of a second station, which is at point Y. A stranded boat at point Z lies out on the ocean between the two stations. If $m\angle YXZ = 48^\circ$ and $m\angle XYZ = 36^\circ$, which station is closer to the stranded boat? What is its distance to the boat?

9. Given a triangle with sides 6 cm and 9 cm, provide a measure for a "third" side that would be impossible to sketch. Use the Law of Cosines to confirm that it is impossible to draw this triangle. In your discussion, be sure to explain why the Law of Cosines confirms that this triangle is not possible.

12. Chris Sollars hits his tee shot. The ball winds up exactly 100 yd from the hole. On his second shot, the ball winds up exactly 50 yd from the hole, somewhere on the circle shown in the figure.



a. If Chris's second shot went on a line 17° to the right of the line to the hole, sketch on the figure the two possible places the ball could be. Calculate the two possible distances the shot could have gone.

b. If Chris's second shot had gone on a line 35° to the right of the line to the hole, show by calculation that it could not have been within 50 yd of the hole. Sketch the path of the ball, illustrating your answer.

c. What is the maximum angle at which Chris could have hit his second shot and still have it come to rest 50 yd from the hole?

Instructional Strategies:

- Daily conversational format through which students will share and debate solutions and concepts
- Use of available technology such as Desmos and the graphing calculator to explore the components of each function and compare the graph to other functions
- Formative assessments throughout to assess current levels of understanding
- Summative assessments will be administered at the end of each unit of study

	Interdisciplinary Connections Students will use the oscilloscope to gather data on the sound waves of two notes played simultaneously. While creating a mathematical model from the given data, students will compare the relevancy between the mathematical exploration and the physics of the sound waves. Technology Integration Students will use an oscilloscope to gather data on the sound waves of two notes played simultaneously. They will then use the data to write the combined sinusoid equation to model the combination of notes and identify the notes that were played Media Literacy Integration Students will identify one example of the use of a trigonometric curve in the media. The summary will include the meaning of the amplitude and period in their example. Students will provide an opinion on whether they feel the graph accurately supports the contents of the report. Global Perspective Students will research sunrise and sunset times for two selected regions of their choice outside of the United States. They will be invited to choose one region close to the Equator and a second region further north. They will use the given data to write a cosine function that will model their data for each region. Their summary will include a discussion about which values in their functions are significantly different and why those differences exist (longer day of sunlight versus shorter day of sunlight and why)

Unit 5 - Vectors

<table>
<tr><th colspan="2">Anchor Standards:

Vector and Matrix Quantities (N-VM)</th></tr>
<tr><td colspan="2">Big Ideas: Course Objectives / Content Statement(s)

• Students will learn how vectors can model physical phenomena.
• Students will learn how to perform operations and computations, and interpret the results of these computations, on vectors in 2 and 3 dimensions.</td></tr>
<tr><th>Essential Questions

What provocative questions will foster inquiry, understanding, and transfer of learning?</th><th>Enduring Understandings

What will students understand about the big ideas?</th></tr>
<tr><td>What natural phenomenon can be modeled using vectors?

How are vectors and vector operations used to represent quantities in real life?

How can vectors be used to describe planes and other 3-dimensional objects?</td><td>Students will understand that…

The students will learn that a vector is used to represent quantities that involve both magnitude and direction. The students will understand how vectors are used to model real life situations, especially in physics problems involving finding force on an inclined ramp or a wind-adjusted bearing in airplane navigation.

Students will be able to apply vector operations of addition, multiplication, scalar multiplication, and dot product. They will be able to identify which operation to use in real life situations.

The cross-product of two vectors creates a normal vector in either direction. Planes and other geometric objects in space can be generated using this simple fact.</td></tr>
<tr><th>Areas of Focus: Proficiencies

(Cumulative Progress Indicators)</th><th>Examples, Outcomes, Assessments</th></tr>
<tr><td>Students will:

▪ (N-VM-1) Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors</td><td>Instructional Focus:

SWBAT:

• Apply all operations on given vectors in both Two- and Three-Dimensions
• Know the difference between a scalar quantity and a vector quantity</td></tr>
</table>

and their magnitudes (e.g., v, |v|, ||v||, v). ▪ (N-VM-2) Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point. ▪ (N-VM-3) Solve problems involving velocity and other quantities that can be represented by vectors. ▪ (N-VM-4) Add and subtract vectors. ▪ (N-VM-5) Multiply a vector by a scalar.	• Given a magnitude and direction, provide the component form of a vector • Given the component form of a vector, provide its magnitude and direction • Know the differences and similarities between a position vector and a displacement vector • Find the unit vector for a given vector • Apply the use of vectors when solving real-world examples using navigational techniques such as "bearing" • Calculate the dot product, scalar projection, and vector projection of a pair of given vectors • Apply the use of vector projections when solving work and force real-world applications • Calculate the cross product between two given vectors • Given two vectors, write the equation of the plane that contains the two vectors • Given two vectors, calculate the area of the parallelogram or triangle formed by the two vectors • Given a pair of vectors in three-dimensions, calculate the direction angles and the direction cosines • Write the vector equation of a line in space (optional topics) Sample Assessments: 1) Find the component form and magnitude of the vector **v** that has the initial point (-2, 3) and terminal point (-7, 9). 2) Let **u**=<1, 2> and **v**=<3, 1>. Find each of the following vectors: a) **u**+**v** b) **u**-**v** c) 2**u**-3**v** 3) Find a unit vector **u** in the direction of **v**=<7, -3> and verify that the result has magnitude 1. 4) Find the direction angle: **v**=-6**i**+6**j** 5) Find the dot product: <3, 4>•<2, -3> 6) Find the angle between **u**=<3, 0> and **v**=<1, 6>. 7) To slide an object across a floor, a person pulls a rope with a constant force of 25 pounds at a constant angle of 30 degrees above the horizontal. Find the work done if the object is dragged 40 feet. 8) The vector $\mathbf{u} = \langle 4000, 5700 \rangle$ gives the number of units of two models of laptops produced by a company. The vector $\mathbf{v} = \langle 1550, 1300 \rangle$ gives the prices (in dollars) of the two models of laptops,

respectively. Identify the vector operation used to increase revenue by 5.5%.

9) A force of 50 pounds is exerted along a rope attached to a crate at an angle of $60°$ above the horizontal. The crate is moved 30 feet. How much work has been accomplished?

10) Eric is sitting on a wagon on the side of a hill inclined at . The combined weight of Eric and the wagon is 160 pounds. What is the magnitude of the force required to keep Eric from sliding down the hill?

11) Find the equation of the plane containing (2, 0, 8) , (4 , 3, 0), and (7 , 5, 1).

12) Given a vector $\vec{v} = 5\vec{i} + 6\vec{j} + 2\vec{k}$, find its direction cosines. Which direction angle can you use to find the angle of elevation? Find the angle of elevation. Find the azimuth angle for this vector.

Instructional Strategies:

- Daily conversational format through which students will share and debate solutions and concepts
- Use of available technology such as Desmos and the graphing calculator to explore the components of each function and compare the graph to other functions
- Formative assessments throughout to assess current levels of understanding
- Summative assessments will be administered at the end of each unit of study

Interdisciplinary Connections

Students will discuss how vectors are used in physics through many different applications involving velocity, force, tension, and work problems. The students will also discuss the use of vectors in navigation problems.

Technology Integration

The students will use the following applet to visualize a resultant vector.

https://phet.colorado.edu/en/simulation/vector-addition

Global Perspectives

The students will research a specific real world application of vectors to share with the class. One example includes selecting a well-known domestic or international airport and examining the effects of wind on the flight of airplanes trafficking that airport. Factors

	should include wind direction and speed and the impact on the flight path of the planes arriving to and departing from the airport.

Unit 6 – Conic Sections

Anchor Standards: Expressing Geometric Properties with Equations (G-GPE)	
Big Ideas: *Course Objectives / Content Statement(s)* • Students will learn and write the standard form equations for circles, parabolas, ellipses, and hyperbolas. • Students will identify the important and defining geometric characteristics of each conic section. • Students will be able to represent conic sections using Cartesian equations and parametric equations using trigonometry.	
Essential Questions *What provocative questions will foster inquiry, understanding, and transfer of learning?*	**Enduring Understandings** *What will students understand about the big ideas?*
What are the geometric definitions of conic sections based on?	Students will understand that… The four conic sections can be generated by slicing a cone with a plane. In addition, they can all be defined as a locus of points with different conditions based on distance from fixed points and/or lines.
What information can we learn about a conic section from its standard form?	The center, foci, directrices, vertices, eccentricity, and other critical features of conic sections can be determined by a few simple parameters in the conic's equation.
How are conic sections related to trigonometry?	The equations for conic sections can be written in parametric form using the trigonometric functions.
Areas of Focus: Proficiencies **(Cumulative Progress Indicators)**	**Examples, Outcomes, Assessments**
Students will: ▪ (G-GPE-1) derive the equation of a circle given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation	Instructional Focus: SWBAT: • Demonstrate how conic sections are formed • Identify the type of conic section based on the given equation • Convert the general equation of a given conic section into its standard form

▪ (G-GPE-2) derive the equation of a parabola given a focus and directrix ▪ (G-GPE-3) derive the equations of ellipses and hyperbolas given the foci, and using the fact that the sum or differences from the foci is constant	• Convert the standard form of a given conic section into its general form • Be able to write a pair of Parametric Equations for a given conic • Given a pair of Parametric Equations, write the standard form equation of the given conic • Given the standard equation of an ellipse of hyperbola, identify its center, major radius, minor radius, eccentricity, equation of directrices, equations of the asymptotes (for the hyperbola only), focus points, and latus rectum points • Given the standard equation of a parabola, identify the vertex, focus, equation of the directrix, axis of symmetry, and latus rectum points • Understand the graphical relationship between the focus point(s) and the points that lie on the conic section • Provide a detailed graph of a given conic section including all key components • Given the sketch of a conic section, write the equation that will model that conic section • Provide a sketch of quadric surfaces (conic sections rotated about a given axis) whose center is the origin • Provide a sketch of quadric surfaces (conic sections rotated about a given axis) whose center is not the origin • Use composite functions to calculate maximum / minimum area / volume of a figure circumscribed by a given conic • Given a small amount data, identify the type of conic section that has been presented and write an equation for that conic • Given an equation in general form in which "B" is not equal to zero, use the discriminant to identify the type of conic section • Given an equation in general form in which "B" is not equal to zero, identify the angle of rotation of the conic • Given a real-world scenario, use the equation of an appropriate conic section to model the scenario and find a solution Sample Assessments: 1) For the equation below, find the indicated information. a) All 4 radii (the values of a, b, c, d) b) The equations of the directrices c) The eccentricity, e d) The coordinates of the foci e) The coordinates of each latus rectum (use the formula for length of the L.R.) f) On the Graph Paper provided, sketch the complete graph of the conic $\frac{(x-3)^2}{16}+\frac{(y-2)^2}{25}=1$

2) Change the Cartesian Equation into a set of Parametric Equation.

$$\frac{(x-3)^2}{25}+\frac{(y+1)^2}{9}=1$$

3) Change the equation to Standard Form

$$4x^2+6y^2-24x-60y+42=0$$

4) Change each equation to General Form

$$\left(\frac{x-1}{3}\right)^2-\left(\frac{y+3}{5}\right)^2=1$$

5) Find a Cartesian Equation (in either general or standard form) that represents the conic described.

Foci located at (3 , - 6) and (3 , 8) , with $e = \frac{2}{3}$

6) A given satellite dish has a diameter of 20.5 inches and a "bowl" depth of 2.25 inches. According to these measurements, where should the receiving device be placed? "? (Hint: create a convenient coordinate system to investigate the values of the parabola)

7) Provide a sketch for the quadric surface described below and identify its correct name:

Rotate $25(x-2)^2+16(y+1)^2=400$ about the line $y = -1$

8) Given each conic, determine the type of conic (be sure to show your calculations)

a. $8x^2-24xy+12y^2-7x+3y-45=0$

b. $12x^2+14xy-5y^2+36x+19y+52=0$

9) Given the equation, $10x^2-22xy+18y^2-6x-14y+68=0$, find the angle (in degree mode) of rotation of the horizontal axis.

Instructional Strategies:

- Daily conversational format through which students will share and debate solutions and concepts
- Use of available technology such as Desmos and the graphing calculator to explore the components of each function and compare the graph to other functions
- Formative assessments throughout to assess current levels of understanding

	• Summative assessments will be administered at the end of each unit of study Interdisciplinary Connections Students will research the use of conic sections in structures throughout history. Potential topics will include arches, archways, circular/spherical structures, elliptical structures. Students will report on the mathematical and historical significance of the structure including the time period when they were created and once significant event that occurred during that time period. A second, more specific option would be to explore the history of the discovery of the rotation of planets and the discovery that these paths are elliptical. Technology Integration Students will use the graphing calculator and Desmos to explore the graphical properties of the conic sections being explored. Media Literacy Integration Students will view the following video: https://www.youtube.com/watch?v=4KHCuXN2F3I Students will be asked to explain the mathematical principles behind the construction of the elliptical pool table Global Perspectives Students will research the use of conic sections in structures throughout history. Potential topics will include arches, archways, circular/spherical structures, elliptical structures. Students will report on the mathematical and historical significance of the structure including the time period when they were created.

Unit 7: Polar Coordinates and Complex Numbers

Anchor Standards: The Complex Number System (N-CN)	
Big Ideas: *Course Objectives / Content Statement(s)* • Students will identify and write equations in polar form for curves and conic sections. • Students will learn to express complex numbers in polar form.	
Essential Questions *What provocative questions will foster inquiry, understanding, and transfer of learning?*	**Enduring Understandings** *What will students understand about the big ideas?*
	Students will understand that…
How can points in space be identified other than indicating horizontal and vertical motion from the origin?	Points in space can be uniquely identified by the distance from the origin and the angle from the positive x-axis.
How can the polar and rectangular systems be exchanged for a given curve?	A simple set of identities, based on the definition of the trigonometric functions, will allow students to translate from rectangular to polar equations, and vise versa.
How are complex numbers plotted? Why is using a polar format logical?	Complex numbers are plotted with the real value on the x-axis and the imaginary value on the y-axis. Complex numbers can be written in "trigonometric form" by simply substituting in the polar coordinates of the point.
Areas of Focus: Proficiencies **(Cumulative Progress Indicators)**	**Examples, Outcomes, Assessments**
Students will:	Instructional Focus:

- (N-CN-1) Know there is a complex number *i* such that $i^2 = -1$, and every complex number has the form a + bi with a and b real.
- (N-CN-2) Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.
- (N-CN-3)Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers.
- (N-CN-4)Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers), and explain why the rectangular and polar forms of a given complex number represent the same number.
- (N-CN-5)Represent addition, subtraction, multiplication, and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation. For example, $(-1 + \sqrt{3}\,i)3 = 8$ because $(-1 + \sqrt{3}\,i)$ has modulus 2 and argument 120°.

SWBAT:

- Use given polar coordinates to plot points on a polar graph
- Given a point of the polar graph, identify the coordinates of that point
- Given a point, provide alternative polar coordinates for the location of that point. There are an infinite number of possibilities, but students should identify at least one in each of the following formats: (+ , +), (+ , -), (- , -), and (- , +)
- Convert from Cartesian Coordinates to Polar Coordinates and from Polar Coordinates to Cartesian coordinates
- Convert a given equation from cartesian form into Polar form or from Polar form into Cartesian form
- Identify the type of polar graph given based on the equation of the graph
- Graph a polar curve given its equation
- When given a system of polar equations, identify true points of intersection versus false points of intersection
- Solve a system of polar equations both algebraically (when possible) or graphically using Auxillary Cartesian equations
- Convert complex numbers in Cartesian form into Polar form
- Calculate the product, quotient or reciprocal of complex numbers in Polar form
- Use DeMoivre's Theorem to find the roots of complex numbers in Polar form
- Use Parametric Equations to find the equations of a Cycloid (this topic is optional)
- Use Parametric Equations to find the equations of moving objects and solve real-world applications (this topic is optional)

Sample Assessments:

1) Write the complex number z=6-6i in trigonometric form.

2) Let $z_1 = 3\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)$ and $z_2 = 4\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)$. Find the product $z_1 z_2$ and the quotient $\frac{z_1}{z_2}$ of the complex numbers.

3) Ex: Use DeMoivre's Theorem to find the roots of $\left(1+i\right)^6$

4) For each polar equation below, identify the type of graph it represents and describe the sketch of the equation. Include as many details as possible.

a. $r = 6 \cos 2\theta$

b. $r = 4 \sin 5\theta$

c. $r = 10 \sin \theta$

d. $r = 8 \csc \theta$

e. $r = 9\sec\theta$

f. $r = 4 + 3\sin\theta$

g. $r = \dfrac{6}{2 + 7\cos\theta}$

h. $r = \dfrac{4}{8 + 8\cos\theta}$

i. $r = 12 + 12\cos\theta$

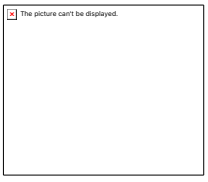
j. $r = \dfrac{5}{8 + 2\sin\theta}$

5) For each set of coordinates, provide three alternative sets of coordinates to represent that point. When finished, you should have four pairs of coordinates – one of each of the following patterns: (+ , +) (+ , -) (- , +) and (- , -)

a) $\left(4, 124^\circ\right)$ b) $\left(-5, 227^\circ\right)$

c) $\left(8, -110^\circ\right)$ d) $\left(-7, -304^\circ\right)$

6) Solve each system algebraically. You can verify you results by using your graphing calculator.



7) For each of the following equations, find an equivalent equation in rectangular coordinates.

a. $r = 2\sin\theta$ b. $r = \dfrac{2}{1 - \cos\theta}$

8) For each of the following equations, find an equivalent equation in polar coordinates.

	a. $x + y = 4$ b. $x^2 + y^2 = 16$ Instructional Strategies: • Daily conversational format through which students will share and debate solutions and concepts • Use of available technology such as Desmos and the graphing calculator to explore the components of each function and compare the graph to other functions • Formative assessments throughout to assess current levels of understanding • Summative assessments will be administered at the end of each unit of study Interdisciplinary Connections Students will explore and gather examples in which polar graphs are used in art and photography. The students will research how complex numbers relate to the laws of electricity. Technology Integration Students will use the graphing calculator or Desmos to explore the graphs of polar equations. Media Literacy Integration Students can explore ways in which imaginary numbers and polar curves are used in the advertising industry. Global Perspectives The students will research to solve the following problem: The famous formula $e^{a+bi} = e^a(\cos b + i \sin b)$ is called Euler's Formula, after the Swiss mathematician Leonhard Euler (1707-1783). This formula gives rise to the equation $e^{\pi i} + 1 = 0$. This equation relates the five most famous numbers in mathematics: 0, 1, π, e, and i in a single equation. Write a paragraph summarizing your findings.

Unit 8: Discrete Math and an Introduction to Calculus

Anchor Standards:

Arithmetic with Polynomials (A-APR)

Big Ideas: *Course Objectives / Content Statement(s)*

- Students will use explicit and recursive equations to analyze finite and infinite sequences and series.
- Students will explore the potential convergence of infinite series.
- Students will be introduced to the concepts of limits and instantaneous rate of change.

Essential Questions *What provocative questions will foster inquiry, understanding, and transfer of learning?*	**Enduring Understandings** *What will students understand about the big ideas?*
	Students will understand that…
How do you find the *n*th term or partial sum of an arithmetic sequence?	The nth term of a sequence may be able to be represented explicitly or recursively. Partial sums of arithmetic sequences can be found by adding the outermost terms and counting the number of such pairs in the sequence.
How do you find terms and sums of geometric sequences?	The students will understand how the formulas for finding *n*th terms and sums of geometric sequences and series are derived, and be able to use them to solve problems.
What patterns can you observe in the expansion of a binomial $(x+y)^n$?	The students will understand the connection between a binomial expansion and Pascal's Triangle. They will learn the formula for binomial coefficients.
What do a convergent infinite geometric series and the derivative of a function at a point have in common?	Both convergent infinite geometric series and the derivative of a function at a point requires the computation of a limit.
Areas of Focus: Proficiencies **(Cumulative Progress Indicators)**	**Examples, Outcomes, Assessments**
Students will:	Instructional Focus:
	SWBAT:
▪ (A-SSE-4) Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems. ▪ (A-APR-5) Know and apply the Binomial Theorem for the	• Identify a given sequence as Arithmetic, Geometric, or neither • Write the explicit formulas for a given sequence • Write as recursive formula for a given sequence • Find the value of a given term within a sequence • Given a term from a given sequence, identify the position number of that term • Calculate the sum of a given series

expansion of $(x+y)^n$ in powers of x and y for a positive integer n, where x and y are any numbers, with coefficients determined for example by Pascal's Triangle. ▪ (F-BF-2) Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.	• Determine if the sum of a given series is either convergent or divergent. • Calculate the sum of a convergent series • Apply the use of either the explicit or recursive definition of a sequence or series to model a given real-world scenario • Use Sigma notation to represent a series • Given Sigma notation, calculate the sum of the given series • Apply the Binomial Theorem to expand a binomial expression raised to a power • Apply the Binomial Theorem to find a particular term within the expansion of a binomial raised to a power • Apply the Binomial Theorem to determine the probability of a particular outcome happening over a given number of experiments • Find the zeros of a function graphically • Use Synthetic Division to find the zeros of a given function • Apply and explain the Remainder and Factor Theorems • Use the Sum and Products of the Zeros of a function to write the particular equation of a function • Given a set of data for a real-world scenario, determine a function of best fit that will model that scenario • Given a rational function, determine whether the function has a removable discontinuity or an asymptote • Understand the difference between a function that is in indeterminate form and one which is in infinite form • Understand the basic definition of a limit • Understand and apply a limit as x approaches from the left and/or as x approaches the limit from the right • Understand when a limit exists and be able to calculate its value • Given a rational function, resolve the function into partial fractions (this topic is optional) • Calculate the Average Rate of Change between two given points • Calculate the Instantaneous Rate of Change at a particular location given one point and second arbitrary point, (x, f(x)) • Calculate the Instantaneous Rate of Change anywhere on the function using the arbitrary points (x, f(x)) and (x + h, f (x + h)) • Use the "f(x +h)" definition of a limit to find the derivative of f(x). • Find the equation of the line tangent to a curve a given point • Apply the rule for the derivative of a polynomial to find the derivative of that function • Use the derivative of a function to prove the location of relative maximum / minimum values of a function • Given the derivative of a function, find the original function (this topic is optional)

Sample Assessments:

1) Find the *n*th term of the arithmetic sequence whose common difference is 5 and whose first term is -1.

2) Ex: Find a formula for the *n*th term of the geometric sequence 6, -2, $\frac{2}{3}$,… What is the tenth term?

3) Find the coefficient of the term a^6b^5 in the expansion of $(2a-5b)^{11}$.

4) Ex: Expand and simplify $(3-2i)^6$.

5) A waiter knows from experience that 7 out of 10 people who dine alone will leave a tip. Tuesday evening, the waiter served 12 lone diners. What is the probability that the waiter received a tip from 9 of these diners?

6) Evaluate each expression below:

a. $\sum_{n=2}^{20} 3n+2$ b. $\sum_{n=1}^{8} 4(3)^n$

7) Ex: Use a table to estimate $\lim_{x\to 0} \frac{x}{\sqrt{x+1}-1}$.

8) Ex: Find each limit: a) $\lim_{x\to 2} x^3$ b) $\lim_{x\to 4} 8x$ c) $\lim_{x\to 16} \sqrt[4]{x}$

9) Ex: Find the limit: $\lim_{x\to 2} \frac{x^2+2x-8}{x-2}$

10) Ex: Find the limit: $\lim_{x\to 0} \frac{\sqrt{x+9}-3}{x}$

11) Ex: Find the slope of the graph of $f(x)=x^3$ at the point (2, 8).

12) Ex: Find the derivative of $f(x)=4x^2-5x$

13) Given the function, $f(x) = 2x^3 - 25x^2 + 97x - 116$, complete the following:

a. Find the average rate of change from $x = 3$ to $x = 3.001$
b. Algebraically, find the instantaneous rate of change at $x = 3$
c. Use the Power Rule to find the general form of , then find . Does it agree with your answer to letter *b*?
d. Use the definition to find the general form of . Does it agree with the expression you found in letter *c*?
e. Find the equation of the line tangent to $f(x)$ at $x = 3$
f. Find the value(s) of x where an extreme value exists
g. Use the values of to determine whether each extreme value is a maximum or a minimum. Explain your answer.

14) An open-top box is to be made from a 24 in. by 36 in. piece of cardboard by removing a square from each corner of the box and folding up the flaps on each side. What size square should be cut out of each corner to get a box with the maximum volume?

Instructional Strategies:

- Daily conversational format through which students will share and debate solutions and concepts
- Use of available technology such as Desmos and the graphing calculator to explore the components of each function and compare the graph to other functions
- Formative assessments throughout to assess current levels of understanding
- Summative assessments will be administered at the end of each unit of study

Interdisciplinary Connections

Students will research the historical timeline highlighting the rise of the study of calculus emphasizing the findings of Newton and Leibnitz. They will make note of the overall culture at the time of these discoveries and the level of importance given to these discoveries in comparison to today's mathematics world.

Technology Integration

	The students will use list editor and sum functions on the graphing calculators to verify the series formulas that they derive. Students will use graphing calculators and Desmos to explore and confirm the values of limits Students will use graphing calculators and Desmos to explore and confirm the equation of lines tangent to a curve at a given point Students will use graphing calculators and Desmos to explore and confirm the location of relative maximum and minimum values Media Literacy Integration The students will choose two different banks, then research information about the Certificate of Deposit accounts they offer. They will highlight important information in the account Terms and Conditions, then calculate the account balance if $5000 is deposited and left in the account for 2 years. Global Perspectives Mathematics History- The students will learn about the mathematician Carl Friedrich Gauss. He famously "invented" the formula for finding the sum of an arithmetic sequence when he was 10 years old. In addition, students will explore the timeline and mathematicians behind some of the most significant mathematical discoveries/ theories. An emphasis will be placed on how globally widespread the study of mathematics and its contributors has been throughout history.

Supports for English Language Learners		
Sensory Supports	**Graphic Supports**	**Interactive Supports**
Real life objects	Charts	In pairs or partners
Manipulatives	Graphic Organizers	In triads or small groups
Pictures	Tables	In a whole group
Illustrations, diagrams & drawings	Graphs	Using cooperative group

Magazines & Newspapers	Timelines	Structures
Physical activities	Number lines	With the Internet / Software
Videos & Film		In the home language
Broadcasts		With mentors
Models & Figures		

Intervention Strategies		
Accommodations	Interventions	Modifications
Allow for verbal responses	Multi-sensory techniques	Modified tasks/expectations
Repeat/confirm directions	Increase task structure (e.g. directions, checks for understanding, feedback	Differentiated materials
Permit response provided via computer or electronic device	Increase opportunities to engage in active academic responding	Individualized assessment tools based on student need
Audio Books	Utilize pre reading strategies and activities previews, anticipatory guides, and semantic mapping	Modified assessment grading

Summit Public Schools

Summit, New Jersey

Curricular Addendum

Career-Ready Practices **CRP1**: Act as a responsible and contributing citizen and employee. **CRP2**: Apply appropriate academic and technical skills. **CRP3**: Attend to personal health and financial well-being. **CRP4**: Communicate clearly and effectively and with reason. **CRP5**: Consider the environmental, social and economic impacts of decisions. **CRP6**: Demonstrate creativity and innovation. **CRP7**: Employ valid and reliable research strategies. **CRP8**: Utilize critical thinking to make sense of problems and persevere in solving them. **CRP9**: Model integrity, ethical leadership and effective management. **CRP10**: Plan education and career paths aligned to personal goals. **CRP11**:. Use technology to enhance productivity. **CRP12**: Work productively in teams while using cultural global competence.	**Interdisciplinary Connections** ● Close Reading of works of art, music lyrics, videos, and advertisements ● Use Standards for Mathematical Practice and Cross-Cutting Concepts in science to support debate/inquiry across thinking processes **Technology Integration** Ongoing: ● Listen to books on CDs, Playaways, videos, or podcasts if available. ● Use document camera or overhead projector for shared reading of texts. Other: ● Use Microsoft Word, Inspiration, or SmartBoard Notebook software to write the words from their word sorts. ● Use available technology to create concept maps of unit learning.

Instructional Strategies:
Supports for English Language Learners:

Sensory Supports	Graphic Supports	Interactive Supports
Real-life objects (realia)	Charts	In pairs or partners
Manipulatives	Graphic organizers	In triads or small groups
Pictures & photographs	Tables	In a whole group
Illustrations, diagrams, & drawings	Graphs	Using cooperative group
Magazines & newspapers	Timelines	structures
Physical activities	Number lines	With the Internet (websites) or software programs
Videos & films		In the home language
Broadcasts		With mentors
Models & figures		

from https://wida.wisc.edu

Media Literacy Integration

- Use multiple forms of print media (including books, illustrations/photographs/artwork, video clips, commercials, podcasts, audiobooks, Playaways, newspapers, magazines) to practice reading and comprehension skills.

Global Perspectives

- The Global Learning Resource Library

Differentiation Strategies:

Accommodations	Interventions	Modifications
Allow for verbal responses	Multi-sensory techniques	Modified tasks/ expectations
Repeat/confirm directions	Increase task structure (e.g., directions, checks for understanding, feedback)	Differentiated materials
Permit response provided via computer or electronic device	Increase opportunities to engage in active academic responding (e.g., writing, reading aloud, answering questions in class)	Individualized assessment tools based on student need
Audio Books	Utilize prereading strategies and activities: previews, anticipatory guides, and semantic mapping	Modified assessment grading