Summit Public Schools
Summit, New Jersey
Grade Level / Content Area: 11-12 / Mathematics
Length of Course: 1 Year

AP Calculus AB

Course Description:

AP Calculus AB is a college-level calculus course that addresses the concepts, mechanics, and applications of limits, derivatives, and integrals. The calculus that students will be exposed to in this course will address functions that are represented algebraically, numerically, graphically, and verbally. The instructor will encourage the connections between these representations throughout the course. Students will make presentations, work cooperatively and communicate mathematically orally and in written form. Technology will be used extensively throughout the course. Students will be expected to use the calculator as a tool to answer conceptual problems that will often involve complicated functions. The standards that follow are from the College Board's course description. These standards are accepted by universities worldwide for college credit.

NOTE:

- The chapters listed below in "Course Pacing" and the individual sections listed under "Instructional Focus" for each standard provide alignment to the required text by Paul Foerster.

- Relevant past AP questions are listed at the end of each standard's "Sample Assessments". These questions are selected from past exams that the College Board has released to the public. These exams can be found at http://apcentral.collegeboard.com/.

Course Pacing:

1.	An Introduction to Limits, Derivatives, and Integrals (Chapter 1)	11 days
2.	The Properties of Limits (Chapter 2)	7 days
3.	Derivatives, Antiderivatives, and Indefinite Integrals (Chapter 3)	15 days
4.	Products and Quotients (Chapter 4)	15 days
5.	Definite and Indefinite Integrals (Chapter 5)	18 days
6.	The Calculus of Exponential and Logarithmic Functions (Chapter 6)	14 days
7.	The Calculus of Growth and Decay (Chapter 7)	20 days
8.	The Calculus of Plane and Solid Figures (Chapter 8)	25 days
9.	The Calculus of Motion (Chapter 10)	9 days
10.	AP Exam Review	19 days
11.	Algebraic Techniques for the Elementary Functions (Chapter 9)	26 days

<table>
<tr><th colspan="2">Standard I: Functions, Graphs, and Limits</th></tr>
<tr><td colspan="2">Big Ideas: This standard exposes student to the concept of a limit, which is the basis for the definitions of derivatives and definite integrals in calculus. Students will become familiar with the concept of an infinitesimal quantity. Limits of functions that are represented in algebraic and graphic form will be emphasized. Technology will aide in the discovery of limit existence in interesting functions.</td></tr>
<tr><th>Essential Questions
What provocative questions will foster inquiry, understanding, and transfer of learning?</th><th>Enduring Understandings
What will students understand about the big ideas?</th></tr>
<tr><td>- What are limits, and why are they important to the study of calculus?

- How can limit notation be used to better describe infinite behavior of functions?

- What does it mean for a function to be continuous?

- How does limit existence affect continuity of a function?</td><td>Students will understand that…

- a limit is the value that a function approaches as the input approaches a particular value.

- limits can be used to describe rates of growth of functions, asymptotic behavior of functions, and the behavior of functions at points of discontinuity.

- for a function to be continuous at a point, its limit as the input approaches that point must equal the function value at that point.

-continuity of a function at a point requires the limit of the function at that point to exist.</td></tr>
<tr><th>Areas of Focus: Proficiencies
(Cumulative Progress Indicators)</th><th>Examples, Outcomes, Assessments</th></tr>
<tr><td>Students will:</td><td rowspan="9">Instructional Focus: (Corresponding Textbook Sections)

1. The Limit of a Function (1-5)
2. Numerical Approach to the Definition of a Limit (2-1)
3. Graphical and Algebraic Approaches to the Definition of a Limit (2-2)
4. The Limit Theorems (2-3)
5. Continuity (2-4)
6. Limits Involving Infinity (2-5)
7. The Intermediate Value Theorem (2-6)
Sample Assessments:</td></tr>
<tr><td>Analyze graphs with the aid of technology.</td></tr>
<tr><td>Use analytic and geometric information to predict and explain the local and global behavior of a function.</td></tr>
<tr><td>Have an intuitive understanding of the limiting process, including one-sided limits.</td></tr>
<tr><td>Calculate limits using algebra.</td></tr>
<tr><td>Estimate limits from graphs or tables of data.</td></tr>
<tr><td>Understand asymptotes in terms of graphical behavior</td></tr>
<tr><td>Describe asymptotic behavior in terms of limits involving infinity</td></tr>
<tr><td>Compare relative magnitudes of functions</td></tr>
</table>

and their rates of change	a. For $f(x)=3x-2$, evaluate f(4), evaluate $\lim_{x\to 4} f(x)$. How close to 4 must you keep x in order for f(x) to stay within 0.6 units of 10? State the formal definition of a limit, and identify the specific values of every parameter in the definition based on the previous question.
Have an intuitive understanding of continuity	b. For $f(x)=x\sin\left(\frac{1}{x}\right)+2$, estimate the value of $\lim_{x\to 0} f(x)$ using your TI graphing calculator. Examine both a table of values and a graph of f(x).
Use the Intermediate Value Theorem and the Extreme Value Theorem to gain a geometric understanding of graphs of continuous functions	c. For $f(x)=1-4\sqrt[3]{7-x}$, at x=6 calculate algebraically a positive value of δ for any E>0.
Understand continuity in terms of limits	d. Evaluate $\lim_{x\to 5}\frac{x^2+3x-40}{x-5}$
	e. Find the value of k such that f(x) is continuous. Let $f(x)=\begin{cases}-0.4x+2, x\le 1\\ 0.3x+k, x>1\end{cases}$
	f. A searchlight shines on a wall. The perpendicular distance from the light to the wall is 100 ft. How close to 90 degrees must the angle be in order for the length of the beam to be at least 1000 feet? Write your answer in terms of the definition of an infinite limit. (Hint, use the limit as the angle approaches 90 degrees.
	g. Relevant AP Questions: i. Open Ended: 2011 AB 6, 2011 AB 2 (Form B) ii. Multiple Choice: 2003 AB 3, 6, 79
	Sample Project:

	Students will be asked to create a display showing a function with either a jump, removable, or infinite discontinuity. The display must contain algebraic, tabular, and graphic analysis of the function's limit and the function's continuity at the point of discontinuity. The display should be made using technology. The use of Geogebra software (www.geogebra.org) will be encouraged. Instructional Strategies: Interdisciplinary Connections In this early part of the course, students will be constantly exposed to functions that model real-world phenomena. Some examples include trigonometric models for biological processes that are periodic, exponential models for economic and sociological phenomena, and polynomial models for functions representing average cost. Students will be expected to use limits and appropriate limit notation to effectively describe the behavior or such models. Technology Integration • Students will be made aware that although many examples of evaluating limits provided in textbooks are solvable using pencil and paper, most models used in real life are not as convenient. Students will be encouraged to use technology as a means of analyzing these complex models. • The TI-83 and/or TI-89 calculators will aide students in evaluating the limit of a function by quickly creating tables of values and by quickly graphing functions at points of interest. It is important to discourage students from using the "trace" feature other than for very

	informal (or very obvious) investigations. • http://www.geogebra.org/ The above link provides free software that students can use to graph functions, solve equations, and evaluate limits with. The software can be downloaded or used on the web. • http://www.wolframalpha.com The above link provides specific mathematical information, based on a simple search engine. The software is free on the web, and can also be downloaded on smartphones as an app for a small cost. Media Literacy Integration Students will be asked to find examples of long-term projections in the news. An in-class discussion can contrast these models with the models studied in class, as well as the relationship between long-term projections and limits at infinity. Global Perspectives Students can investigate the development of the limit as a more "modern" way to solidify concepts in calculus that pre-date the limit. The concept of a limit can be found in many cultures' mathematical history, and is a very interesting part of the history of calculus.
The following skills and themes listed to the right should be reflected in the design of units and lessons for this course or content area.	21st Century Skills: Creativity and Innovation Critical Thinking and Problem Solving Communication and Collaboration Information Literacy

	Media Literacy Life and Career Skills 21st Century Themes (as applies to content area): Financial, Economic, Business, and Entrepreneurial Literacy Civic Literacy Health Literacy

Standard II: Derivatives	
Big Ideas: Students should understand the meaning of the derivative in terms of an instantaneous rate of change of a function at a point. Students should understand the derivative function as a function that measures this rate of change given any input. The derivative should be understood for functions represented graphically, numerically, analytically, or verbally. Both algebra and technology will be used to answer questions about the behavior of functions.	
Essential Questions *What provocative questions will foster inquiry, understanding, and transfer of learning?*	**Enduring Understandings** *What will students understand about the big ideas?*
- What is the difference between an average rate of change and an instantaneous rate of change? - What mathematical challenge is encountered when computing an instantaneous rate of change? How can this challenge be resolved? - Do patterns exist that make the computation of derivatives simpler? - In what ways are f, f', and f'' related to each other? - What is the relationship between differentiability and continuity?	Students will understand that… - An instantaneous rate of change takes the time interval to zero. - To compute an instantaneous rate of change, division by zero is required. Derivatives are defined to be the limit of the average rates of change as the time interval *approaches* zero. - Shortcuts such as the power, product, quotient, and chain rules exist that make the algebraic computation of derivative functions faster. - Both algebraic and geometric patterns exist between a function and its first and second derivative. - If a function is differentiable at a point, then it must be continuous at that point. The converse of this is not necessarily true.
Areas of Focus: Proficiencies (Cumulative Progress Indicators)	**Examples, Outcomes, Assessments**
Students will:	Instructional Focus: 1. The Concept of Instantaneous Rate (1-1) 2. Rate of Change by Equation, Graph, or Table (1-2) 3. Difference Quotients and One Definition of the Derivative (3-2) 4. Derivative Functions, Numerically and Graphically (3-3) 5. The Derivative of a Power and Another
Understand that a derivative can be presented graphically, numerically, and analytically	
Understand that the derivative can be interpreted as an instantaneous rate of change.	
Understand that the derivative is defined as the limit of the difference quotient.	
Explain the relationship between	

differentiability and continuity.	Definition of the Derivative (3-4) 6. Introduction to Sine, Cosine, and Composite Functions (3-6) 7. Derivatives of Composite Functions – The Chain Rule (3-7) 8. Derivative of a Product of Two Functions (4-2) 9. Derivative of a Quotient of Two Functions (4-3) 10. Derivatives of Other Trigonometric Functions (4-4) 11. Derivatives of Inverse Trigonometric Functions (4-5) 12. Differentiability and Continuity (4-6) 13. Graphs and Derivatives of Implicit Relations (4-8) 14. Linear Approximations and Differentials (5-3) 15. Antiderivative of the Reciprocal Function (6-2) 16. Derivatives of Exponential Functions using Logarithmic Differentiation (6-5) 17. The Derivative of Base b Logarithmic Functions (6-6) Sample Assessments: a. Given the graph of a polynomial, examine the rate of change of the function at particular points. Describe the rate of change as fast or slow and increasing or decreasing. b. Given a table of values, estimate the instantaneous rate of change both at given points and points that are not given. c. Given an algebraic function, estimate the instantaneous rate of change by creating a table in which several average rates of change are computed, narrowing the time interval at each computation. d. Given $f(x)=x^2+6x-2, c=-4$, use the definition of the derivative to evaluate $f'(c)$. e. Prove the power rule, $\frac{d}{dx}x^n = nx^{n-1}$. f. Compute the derivative of the following: ▪ $y=\frac{5}{2}x^3-\frac{1}{3x^2}$ ▪ $y=2\cos(2x)$ ▪ $f(x)=-9\sin^3(3x-2)$ ▪ $g(w)=w^2\sin^2 w$ ▪ $x(t)=\frac{12}{7t^2}$
Interpret the derivative at a point as the slope of a curve at that point.	
Identify points where derivatives do not exist, such as at vertical tangent lines, cusps, and corners.	
Compute the tangent line to a curve at a point and local linear approximations.	
Define the instantaneous rate of change to be the limit of the average rate of change.	
Approximate rates of change from graphs and tables of values.	
Identify corresponding characteristics of the graphs of f, f', and f''.	
Understand the relationship between the concavity of *f* and the sign of *f''*.	
Define points of inflection as places where concavity changes.	
Compute the derivatives of basic functions, including power, exponential, logarithmic, trigonometric, and inverse trigonometric functions.	
Apply derivative rules for sums, products, and quotients of functions.	
Use implicit differentiation to find the derivative of an inverse function	
Compute derivatives using the chain rule and implicit differentiation.	

	▪ $y = \frac{3x-7}{6x+5}$ ▪ $f(x) = \log_4 x$ ▪ $g(t) = -0.4e^{.18t}$ g. Prove, using the quotient rule, $\frac{d}{dx}\sec x = \sec x \tan x$ h. Use implicit differentiation to compute $\frac{d}{dx}\tan^{-1}(3x)$ i. Find a general formula for $\frac{d}{dx}f^{-1}(x)$. j. Write the equation for a function that is continuous at the point (3,9) but is not differentiable at that point. k. Relevant AP Questions: ▪ Open Ended: 2010 AB 2a, 2007 AB 3d, 2010 AB 2a,c (Form B) ▪ Multiple Choice: 2003 AB 1, 4, 7, 9, 13, 14, 16, 24; 1997 AB 76, 79, 80, 86, Sample Project: Students will create their own derivative matching game. Each student individually will create 10 "interesting" graphs. Then, students will pair up and attempt to draw the graph of the derivative function for each "interesting" graph. Once both students agree that both sets of graphs and derivatives are drawn correctly, a final draft of the 10 "interesting" graphs and corresponding derivatives will be drawn on index cards for a matching game. Instructional Strategies: Interdisciplinary Connections Students will be analyzing the derivatives of functions that are mathematical models for scenarios in the social sciences, physics, biology, and economics. Technology Integration • TI graphing calculators will be used extensively to

	create graphs and tables, as well as assist in numerically evaluating the derivative of a function at a point. This skill is critical for success on the AP Calculus AB exam. • http://www.geogebra.org, http://www.wolframalpha.com/ The above websites allow students to graph functions, and then easily graph the derivative of the function or numerically evaluate the derivative of the function at a point. Media Literacy Students will be asked to find instances in the media (print, web, television) of derivatives in the social and physical sciences. Through this, students will become aware of how widespread the notion of a rate of change is in the physical and social sciences. This will also promote the discussion of an average rate of change versus and instantaneous rate of change. Global Perspectives Students can examine the growth rates of different nations. Students will be asked to draw conclusions about how and why different societies' populations have varied growth rates, and what implications these rates have on nations' economies. http://www.un.org/esa/population/ publications/longrange2/WorldPop2300final.pdf The above link provides an example of such a study. Students should be able to identify the population function and growth rate functions' graphs in the document.
The following skills and themes listed to the right should be reflected in the design of units and lessons for this course or content area.	21st Century Skills: Creativity and Innovation Critical Thinking and Problem Solving Communication and Collaboration Information Literacy

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<table>
<tr><th colspan="2">Standard III: Integrals</th></tr>
<tr><td colspan="2">Big Ideas: Students will understand a definite integral to be a limit of a Riemann Sum, or alternately as the net accumulation of change. Students will discover the inverse relationship between the derivative and the definite integral in their study of the Fundamental Theorem of Calculus. Algebraic, geometric, and technological strategies will be used to evaluate definite integrals. Students will also use appropriate notation to communicate an antiderivative as an indefinite integral.</td></tr>
<tr><th>Essential Questions
What provocative questions will foster inquiry, understanding, and transfer of learning?</th><th>Enduring Understandings
What will students understand about the big ideas?</th></tr>
<tr><td>- What is the difference between “total area” and “net area”?

- How can displacement be retrieved from a function that measures velocity?

- How can displacement be geometrically interpreted on a graph of time vs. velocity? How is this different from total distance traveled?

- How can irregular areas be approximated?

- How can this approximation be improved?

- How are definite integrals and derivatives related to each other?

- How can definite integrals be exactly evaluated?</td><td>Students will understand that…

- In calculus, area below the x-axis is represented as negative, due to the negative value of the function bounding the area.

- The product of the width of the time interval and the height of the bounded function gives the displacement. This simple computation is only practical in a constant function, resulting in the area of a rectangle.

- Displacement can be geometrically interpreted as the net area between the velocity curve and the x-axis. The total distance traveled is simply the total area between the velocity curve and the x-axis.

- Irregular areas can be approximated by building either many rectangles or many trapezoids with bases on the x-axis and heights determined by the function values. When using rectangles, this is called a Riemann Sum.

- The approximation is improved by taking the limit of the rectangles in the Riemann Sum to zero.

- The derivative and definite integral are inverse operations:

$$\frac{d}{dx}\int_a^x f(t)\,dt = f(x)$$

- An antiderivative evaluated from the lower to upper bound of the interval provides the exact value of a definite integral.</td></tr>
</table>

- Does a definite integral always represent area?	- The value of definite integral can be interpreted and displayed as area on a rate vs. time graph, but need not represent area.
Areas of Focus: Proficiencies (Cumulative Progress Indicators)	**Examples, Outcomes, Assessments**
Students will:	Instructional Focus: 1. One Type of Integral of a Function (1-3) 2. Definite Integrals by Trapezoids, from Equations and Data (1-4) 3. Antiderivatives and Indefinite Integrals (3-9) 4. Formal Definition of Antiderivative and Indefinite Integrals (5-4) 5. Riemann Sums and the Definition of the Definite Integral (5-5) 6. Some Very Special Riemann Sums (5-7) 7. The Fundamental Theorem of Calculus (5-8) 8. Definite Integral Properties and Practice (5-9) 9. A Way to Apply Definite Integrals (5-10) 10. Antiderivative of the Reciprocal Function (6-2) 11. Natural Logarithms and Another Form of the Fundamental Theorem (6-3) 12. Derivative and Integral Practice for Transcendental Functions (6-9) Sample Assessments: a. Let $v(t)=100(1-0.9^t)$ be the velocity of a sports car. Plot the graph, then estimate the area between the graph and the x-axis on the domain [0, 10]. Interpret the value of this area in the context of the problem. b. Given a table of data, estimate the value of the definite integral using rectangular and trapezoidal approximations. c. Compute the antiderviative of $f'(x)=(3x-1)^2$ d. Evaluate the indefinite integral: a. $\int 9x^{-3}\,dx$ b. $\int \sin r\,dr$ c. $\int 3.4e^{-2x}\,dx$ d. $\int \frac{\ln^2 x}{x}dx$
Define definite integrals as the limit of Riemann Sums.	
Interpret the definite integral of the rate of change of a quantity over an interval interpreted as the change of the quantity over the interval.	
Apply basic properties of definite integrals including additivity and linearity.	
Use the Fundamental Theorem of Calculus to evaluate definite integrals.	
Use the Fundamental Theorem of Calculus to represent a particular antiderviative, and the analytical and graphical analysis of functions so defined.	
Compute antiderivatives following directly from derivatives of basic functions.	
Compute antiderivatives by substitution of variables, including change of limits for definite integrals.	
Use Riemann Sums, using left, right, and midpoint evaluation points, to numerically approximate definite integrals of functions represented algebraically, graphically, and by tables of values.	
Use Trapezoidal Sums to approximate definite integrals of functions represented algebraically, graphically, and by tables of values.	

	e. Estimate $\int_1^2 2^x dx$ using a left Riemann Sum with 4 equal partitions. Is this answer an over or under estimate for $\int_1^2 2^x dx$? Explain. Use www.geogebra.org to compute Riemann Sums as $n \to \infty$. Use these results to predict the exact value of $\int_1^2 2^x dx$. f. If $g'(x) = f(x)$, prove $\int_a^b f(x)dx = g(b) - g(a)$ g. Evaluate $\int_{-1}^{1} \sqrt{4x+5}\, dx$ using the Fundamental Theorem of Calculus. h. Compute $\frac{d}{dx}\int_1^{2x} \sin 2t\, dt$. i. If the force to stretch a spring x inches is given by F=0.6x, use a definite integral to compute the amount of work (in inch-lbs) to stretch the spring from 0 to 9 inches. j. Relevant AP Questions: a. Open Ended: 2010 AB 2c, 5a, 2009 AB 6b (Form B), 2007 AB 2a, 2b, 3c, 2009 AB 2, 5b b. Multiple Choice: 2003 AB 2, 5, 8, 11, 23, 77, 85, 92 Sample Project: In pairs, students will be asked to go on a “road trip” for 15 minutes. One student will drive, and the other will simply record data. At predetermined intervals, the recorder will write down the car’s speed as well as the odometer readings at the beginning and end of the trip. With this discrete data, students will create a table of values from which they will estimate the total distance traveled during their trip. Students will use rectangular and trapezoidal estimates.

	Then, students will use their TI graphing calculator to create a regression equation for their velocity during the trip. The definite integral of this equation on the appropriate time interval will give a different estimate for the total distance traveled. Finally, students will be asked to estimate their acceleration at various times. Instructional Strategies: Interdisciplinary Connections Students will be analyzing the derivatives and integrals of functions that are mathematical models for scenarios in the social sciences, physics, biology, and economics. The relationships between the derivatives and integrals of the different functions will be discussed in the context of the model. Technology Integration • TI graphing calculators will be used extensively to create graphs and tables, as well as assist in numerically evaluating the definite integral of a function on an interval. This skill is critical for success on the AP Calculus AB exam. • http://www.geogebra.org, http://www.wolframalpha.com/ The above websites allow students to graph functions and then easily graph a Riemann or trapezoidal sum with n partitions, or even compute the exact value of a definite integral. Media Literacy Students will be asked to find examples of accumulation functions (functions defined by integrals) that are used as models in the physical sciences. Students will be asked to research academic sources to explain the significance of the function in the particular field of study.

	Global Perspectives Students will research and learn about Zeno's Dichotomy Paradox and examine how it relates to infinitesimal quantities, infinite sums, and definite integrals.
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Standard IV: Applications of Calculus	
Big Ideas: Students will apply the skills learned in Standards I-III, particularly for problems and applications involving changing functions. Geometric problems such as computing area, volume, and optimum dimensions of objects will be emphasized. Students will be asked to solve problems in the physical and social sciences that deal with rates of change and accumulation for mathematical models. These models will be presented in algebraic, graphic, tabular, and written forms. Technology will be used extensively to efficiently solve such problems.	
Essential Questions *What provocative questions will foster inquiry, understanding, and transfer of learning?*	**Enduring Understandings** *What will students understand about the big ideas?*
	Students will understand that…
- How can functions be written that model common rates of change?	- Differential equations are equations that use a derivative as a variable representing a rate of change. The rate of change of special growth patterns such as linear, exponential, and logistic growth can be easily modeled using a differential equation.
- Given an initial condition, how can differential equations be solved to retrieve a particular solution?	- Differential equations can be solved algebraically (separation of variables among other more complicated methods), numerically using Euler's Method, and graphically using slope fields.
- How can differential equations be graphed while considering all possible initial conditions?	- A slope field is a visual representation of the family of solutions to a given differential equation.
- Where do commonly used volume formulas come from (such as volume of cylinder, sphere, cone)?	- Integral calculus can be used to find the volume of any object with geometrically similar cross-sections. These objects have circular cross-sections, making the procedure particularly simple.
- How can extreme values of functions be found using calculus?	- Derivatives can detect the location maximum and minimum values of continuous functions by finding where the slope of the curve is equal to zero.

- How can we find the average of an infinite number of values along a function? - How can the Fundamental Theorem of Calculus be proven?	- Definite integrals provide a method for finding the average value of a function on an interval. - The Mean Value Theorem is critical in executing one elegant proof of the Fundamental Theorem of Calculus, providing the critical link between derivatives and definite integrals.

Areas of Focus: Proficiencies (Cumulative Progress Indicators)	**Examples, Outcomes, Assessments**
Students will:	Instructional Focus: 1. The Mean Value Theorem and Rolle's Theorem (5-6) 2. Exponential Growth and Decay (7-2) 3. Other Differential Equations for Real-World Applications (7-3) 4. Graphical Solutions of Differential Equations by Using Slope Fields (7-4) 5. Numerical Solutions of Differential Equations by Using Euler's Method (7-5) 6. Predator-Prey Population Problems (7-6) 7. Critical Points and Points of Inflection (8-2) 8. Maxima and Minima in Plane and Solid Figures (8-3) 9. Area of a Plane Region (8-4) 10. Volume of a Solid by Plane Slicing (8-5) 11. Volume of a Solid of Revolution by Cylindrical Shells (8-6) 12. Distance, Displacement, and Acceleration for Motion Along a Line (10-2) 13. Average Value Problems in Motion and Elsewhere (10-3) 14. Related Rates (10-4) 15. Minimum Path Problems (10-5) 16. Maximum and Minimum Problems in Motion and Elsewhere (10-6) Sample Assessments: a. Given $f(x) = x^2$, find the value of x=c on [0, 2] that satisfies the conclusions of the mean value theorem. b. Create your own function that does not satisfy the hypotheses of Rolle's Theorem.
Understand and apply the Mean Value Theorem, as well as the "Mean Value Theorem for Integrals"	
Completely analyze the features of a function, including notions of monotonicity and concavity.	
Optimize functions, identifying both absolute and relative extrema.	
Interpret the derivative as a rate of change in varied applied contexts, including velocity, speed, and acceleration.	
Model the geometric interpretation of differential equations via slope fields and the relationship between slope fields and solution curves for differential equations.	
Use integrals appropriately to model physical, biological, or economic situations.	
Solve problems involving integrals with specific applications including finding area of a region, the volume of a solid with a known cross section, the average value of a function, the distance traveled by a particle along a line, and the accumulated change from a rate of change.	
Find specific antiderivatives using initial conditions, including applications to motion along a line.	
Solve separable differential equations and use them in modeling.	

	c. The rate of growth of a species of bacteria is directly proportional to the amount of bacteria present. If there were initially 5 million bacteria and after 3 minutes there are 7 million bacteria, how many bacteria are present after 10 minutes? d. Solve the differential equation: a. $\frac{dy}{dx} = xy$ b. $\frac{dx}{dt} = \frac{x}{t}$ c. $\frac{dW}{dt} = F - kW$ e. Match 5 given differential equations with 5 given slope fields. f. Draw a slope field for $\frac{dy}{dx} = x - 1$. g. Given $f(x) = x^{5/3} + 5x^{2/3}$, identify all critical points, inflection points, intervals increasing/decreasing, and intervals concave up/down. h. Classify the critical points in the example above as local maxima, local minima, or neither. Justify using both the first and second derivative tests. i. A rectangular box with a square base and no top is to be made from a total of 120 square centimeters of cardboard. What are the dimensions of the box with the maximum volume? j. Set up an integral that represents the area of the region bounded by $y = 2e^{0.2x}$ and $y = \cos x$ between 0 and 5. k. The region bounded by $y = 4 - x^2$, $x = 0$, and $y = 0$ is rotated about the line x=3. Find the resulting volume using both the method of washers and cylindrical shells. l. If the velocity of an object is given by $v(t) = t^2 - 10t + 16$, compute the distance traveled and the displacement of the object from t=0 to t=6. Then, compute the object's acceleration at t=2. Finally, compute the average velocity from t=0 to t=6. m. Compute the average value of $h(x) = \tan x$

	on the interval [0.5, 1.5]. n. If a balloon is a perfect sphere and you want the radius to increase at 2cm/sec, how fast must you be blowing air into the balloon when the radius is 3 cm? o. John is on a boat 150 yards from the shore. His house is on the shoreline, 400 yards from the point on the shore perpendicular to the boat. If he travels on the water 4 feet per second but runs 5 feet per second on land, how far down the shoreline should he dock his boat so he makes it home as fast as possible? p. Relevant AP Questions: a. Open Ended Questions: 2010 AB 5b, c, 2006 AB 2 (Form B), 2011 AB 5, 2008 AB 5, 2010 AB 6, 2005 AB 2 (Form B), 2009 AB 1, 2007 AB 4, 2011 AB 3, 2009 AB 4 b. Multiple Choice Questions: 1997 AB 5, 8, 9, 16, 20, 22, 23; 2003 AB 76, 78, 81, 82, 83, 84, 86, 87, 88, 91, 92 Sample Projects: • Students will analyze the packaging of a product. Given a fixed volume and the geometric shape of the packaging, students will compute the optimal dimensions of the packaging that would minimize surface area, and in turn, help to minimize packaging cost. These optimum dimensions will be compared to the actual dimensions of the packaging. Students will be asked to consider logistical restrictions in describing why the company chose to not use the optimal dimensions. • Students will create a 3-dimensional model for a volume with known cross-sections problem that was solve in class or on homework. A sheet of paper or cardboard should hold the 2-dimensional coordinate plane with the bounding functions drawn to scale. Several cross-sections should be attached to the coordinate plane and fastened so that they are perpendicular to

	the axis described in the problem. Instructional Strategies: Interdisciplinary Connections Students will investigate how differential equations are commonly used to model the rate of change of populations within a certain ecosystem (including humans!). Constant, linear, exponential, and logistic growth rates will be examined. Students will be asked to discuss how limiting factors in the environment of the species may or may not inhibit unbounded population growth. Technology Integration • The TI-89 graphing calculator will be used to quickly sketch slope fields for a given differential equation. • Students will learn how to program a spreadsheet in Microsoft Excel to model many computations from Euler's Method. The resulting data points from Euler's Method can be graphed on Excel, creating an approximate solution to a given differential equation. • Geogebra (www.geogebra.org/) and Wolfram Alpha (www.wolframalpha.com/) can be used to graph the slope fields of complex differential equations, as well as find solutions to differential equations given an initial condition. Media Literacy Students will be asked to research the spread of a recent epidemic and discuss whether or not the epidemic followed an exponential, logistic, or different growth rate. Then students will be asked to find a differential equation and particular solution that models the rate of change and the number infected, respectively.

	Global Perspectives The population growth rates for different nations will be used to motivate the topic of differential equations.
The following skills and themes listed to the right should be reflected in the design of units and lessons for this course or content area.	21st Century Skills: Creativity and Innovation Critical Thinking and Problem Solving Communication and Collaboration Information Literacy Media Literacy Life and Career Skills 21st Century Themes (as applies to content area): Financial, Economic, Business, and Entrepreneurial Literacy Civic Literacy Health Literacy

Required Texts and Resources:

Foerster, Paul A. Calculus: Concepts and Applications. Key Curriculum Press, 1998.

TI-83 Plus Graphing Calculator. Texas Instruments.

TI-89 Graphing Calculator. Texas Instruments.