Summit Public Schools
Summit, New Jersey
Grade Level 12 / Content Area: Mathematics
Length of Course: Full Year

Calculus Curriculum

(Adam Leaman 2018)

Course Description: Topics learned in Pre-Calculus will be solidified and extended to reveal their connections to Calculus. The main objective of the course is to explore the concepts of, and connection between limits, derivatives and integrals, with an emphasis on real-world applications in the sciences, business, and economics.

Pacing Guide

Unit 1 – Functions/Continuity/Limits

Topic	Time Frame
1.1 Functions and Change	1
1.2 Exponential Functions	1
1.3 New Functions from Old	1
1.4 Logarithmic Functions	1
1.5 Trigonometric Functions	3
1.6 Powers, Polynomials and Rational Functions	3
Test	1
1.7 Introduction to Continuity	1
1.8 Limits	3
Test Unit 1	1
Misc. Quizzes / Test Review / Extension time for any section (if needed)	3
Total	19 Days

Unit 2 – Introduction to Derivatives

Topic	Time Frame
2.1 How Do We Measure Speed	1
2.2 The Derivative at a Point	2
2.3 The Derivative Function	3
2.4 Interpretations of the Derivative	2
2.6 Differentiability	2
Test Unit 2	1
Misc. Quizzes / Test Review / Extension time for any section (if needed)	3
Total	14 Days

Unit 3 – Derivative Rules

Topic	Time Frame
3.1 Powers and Polynomials	2
3.2 The Exponential Function	2
3.3 The Product and Quotient Rule	2
3.4 The Chain Rule	3
3.5 The Trigonometric Functions	2
3.6 The Chain Rule and Inverse Functions	3
3.7 Implicit Functions	2
4.6 Related Rates	4
Unit Test	1
Misc. Quizzes / Test Review / Extension time for any section (if needed)	3
Total	Days
	24 days

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Unit 4 – Applications of the Derivative

Topic	Time Frame
3.9 Linear Approximation and the Derivative	2
3.10 Theorems About Differentiable Functions	2
2.5/4.1 Using the First and Second Derivative	3
4.2 Optimization 4.3 Optimization and Modeling	3
Optimization Project	4
4.7 L'Hopital's Rule	1
Unit Test	1
Misc. Quizzes / Test Review / Extension time for any section (if needed)	2
Total	22 days

Unit 5 – Introduction to Integrals

Topic	Time Frame
5.1 How do We Measure Distance Traveled?	1
5.2 The Definite Integral	2
7.5 Numerical Methods for Definite Integrals	2
6.2 Constructing Antiderivatives	2
Unit Test	1
Misc. Quizzes / Test Review / Extension time for any section (if needed)	2
Total	10 days

Unit 6 – Integration Techniques/Applications of Integration

Topic	Time Frame
5.3 The Fundamental Theorem of Calculus (part 1)	2
6.1 Constructing Antiderivatives	2
6.3 Differential Equations and Motion	1
6.4 The Second Fundamental Theorem of Calculus	1
7.1 Integration by Substitution	3
8.1 Area and Volume	3
8.2 Applications to Geometry	3
Supplement - Shell Method	3
Unit Test	1
Misc. Quizzes / Test Review / Extension time for any section (if needed)	2
Total	21 days

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Unit 1:
Functions/Continuity/Limits

The connectedness between linear, quadratic, exponential, logarithmic, rational, and trigonometric functions form the foundation for the conceptual understanding of continuity and limits.

Big Ideas:

- Functions can be represented in different ways
- Continuity leads to the concept of a limit
- Limits form the foundation of differential calculus

Essential Questions *What provocative questions will foster inquiry, understanding, and transfer of learning?*	**Enduring Understandings** *What will students understand about the big ideas?*
• What do the characteristics of a function tell us about the behavior of the graph of that function? • How can new functions be made from old, and what changes when that happens? • What does it mean to be continuous? What factors cause a function to be discontinuous? • What is *infinitesimal* and how does this concept relate to continuity and limits? • What does the limit of a function at certain values tell us about the asymptotes of the graph of that function? • What is a limit and how does it help us in other areas of Calculus?	• Modeling real-world data with functions can help predict future behavior • Algebraic operations can be applied to functions (including the composition of functions). • Exponential functions dominate power and linear functions • A continuous function has a graph that can be drawn without lifting the pencil from the paper. • Continuity can be examined from a graph or table. • There are left-sided and right-sided limits. Sometimes, a limit does not exist. • If the limit as x approaches a value is not the same as evaluating the function for the same value, the function is discontinuous at that value. • Dividing by zero can lead to two different types of discontinuities. A function must be completely simplified before deciding. • A vertical asymptote is a type of discontinuity that can occur in the graph of a function • A horizontal asymptote occurs when the limit as x goes to infinity and/or negative infinity approaches a finite number.

Areas of Focus: Proficiencies (Cumulative Progress Indicators)	Examples, Outcomes, Assessments
Students will: • Find domain, range, compositions and inverses of functions • Sketch graphs by applying transformations to the parent function • Analyze the properties of linear, quadratic, exponential, logarithmic, rational and trigonometric functions. • Use graphing technologies (TI-83+ and Desmos online calculator, Geogebra, Wolfram Alpha) to assist in the discovery and identification of limits, discontinuities, end behavior, and other unique characteristics of functions. • Locate and determine the types of discontinuities graphically, algebraically, and by examining tables • Find limits graphically, algebraically and by examining tables. • Use limit notation	Instructional Focus: • 1.1 Functions and change • 1.2 Exponential functions • 1.3 New functions from old • 1.4 Logarithmic functions • 1.5 Trigonometric functions • 1.6 Power, Polynomial and Rational functions • 1.7 Introduction to continuity • 1.8 Limits Sample Assessments: • Classwork and homework • Exit tickets/warm-ups/short quizzes • Long quizzes • Unit exam Summative assessments/projects that encompass all skills learned in this unit can be found on pages 73-74 in the text. These assessments can be given independently or as a group, and include 21st century themes and skills and higher-order thinking to complete. Instructional Strategies: **Interdisciplinary Connections** • Half-life calculations using logarithms and exponential functions (Science) • Modeling supply and demand for products, finding break even points (Business, Economics, Finance) **Technology Integration** • TI 83+, Desmos, Geogebra, Wolfram Alpha will assist in the discovery and identification of limits, discontinuities, end behavior, and other unique characteristics of functions. **Global Perspectives/ Culturally Responsive Teaching** • Population studies from various countries using exponential functions

The following skills and themes listed to the right should be reflected in the design of units and lessons for this course or content area.	21st Century Skills: Creativity and Innovation Critical Thinking and Problem Solving Communication and Collaboration Information Literacy Media Literacy Life and Career Skills 21st Century Themes (as applies to content area): Financial, Economic, Business, and Entrepreneurial Literacy Civic Literacy Health Literacy S.T.E.A.M. (Science, Technology, Engineering, Arts, Mathematics)

Unit 2:
Introduction to Derivatives

Real-world average and instantaneous velocity examples are studied numerically, algebraically and graphically, all leading to the definition of a derivative and requirements for differentiability.

Big Ideas:

- Calculus is the study of change
- Derivatives help us understand events in the sciences and economics.

Essential Questions *What provocative questions will foster inquiry, understanding, and transfer of learning?*	**Enduring Understandings** *What will students understand about the big ideas?*
• How is speed measured? How is it different than velocity? • What is the difference between average velocity and instantaneous velocity? • How are average velocity and instantaneous velocity represented graphically? • How is the slope of a line used to find the derivative of any curve at a point? • Can we find the derivative of an entire function at once, or only one point at a time? • What does the graph of the derivative function look like? • How can derivatives be found from a graph, table of values, or formula? • How do we represent derivatives and what units do we place on them? • What does it mean to be differentiable? • What is the difference quotient and how does it relate to the concept of a limit?	• Average velocity of an object over an interval is the net change in position during the interval divided by the change in time • Instantaneous velocity of an object at a specific time is the limit of the average velocity over an interval, as the interval shrinks around the specific time • Linear functions help us find the instantaneous velocity. The instantaneous velocity is the slope of a curve at a point • If the derivative is positive on an interval, the curve is increasing over that interval, and if the derivative is negative on an interval, the curve is decreasing over that interval • The derivative of a line is the slope of that line • The derivative of a constant is zero • Functions are not differentiable at vertical tangents, sharp corners, or any discontinuity. • dy/dx and f' are common and acceptable ways to represent the derivative

Areas of Focus: Proficiencies (Cumulative Progress Indicators)	Examples, Outcomes, Assessments
Students will: • Calculate average velocity using real-world data from tables, graph, and functions • Given a function f, sketch the derivative graph f' • Estimate instantaneous velocity at a point using information from tables and graphs • Use the definition of a derivative to calculate instantaneous velocity at a point • Use the definition of a derivative to find a derivative function • Sketch graphs based on properties (i.e. Sketch f given that on the interval $0<x<4$, $f'>0$, on all other intervals, $f'<0$). • Interpret statements and scenarios using derivative notation	Instructional Focus: • 2.1 How do we measure speed? • 2.2 The derivative at a point • 2.3 The derivative function • 2.4 Interpretations of the derivative • 2.6 Differentiability Sample Assessments: • Classwork and homework • Exit tickets/warm-ups/short quizzes • Long quizzes • Unit exam Summative assessments/projects that encompass all skills learned in this unit can be found on page 122 in the text. These assessments can be given independently or as a group, and include 21st century themes and skills and higher-order thinking to complete. Instructional Strategies: **Interdisciplinary Connections** • Interpreting rate of change of chemical reactions, temperature (science), interest and principal values in bank accounts (business/finance) **Technology Integration** • TI 83+, Desmos, Geogebra, Wolfram Alpha will assist in the discovery and identification of derivatives **Global Perspectives/ Culturally Responsive Teaching** • Interpreting rate of change of population, birth rates, earthquakes and bodies of water around the world

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Unit 3:
Derivative Rules

Derivative rules are examined and applied using examples that develop students' intuition behind them.

Big Ideas:

- Calculus is the study of change
- Derivatives help us understand events in the natural sciences and economics.

Essential Questions *What provocative questions will foster inquiry, understanding, and transfer of learning?*	**Enduring Understandings** *What will students understand about the big ideas?*
• How are derivatives connected to the functions studied in Unit 1? • Does applying algebraic operations to functions change the derivative of a function? • Is using the definition of a derivative the only way to calculate a derivative? • What is an implicit function and how do you find the derivative of an implicit function? • Can you take the derivative of a derivative? When does it stop?	• Shortcuts can be used to bypass using the definition of a derivative • Taking the derivative of a constant multiplied by a function is the same as taking the derivative of that function and then multiplying by the constant [$d/dx\ [(cf(x))] = c\ f'(x)$] • $d/dx\ [f(x) +- g(x)] = f'(x) +- g'(x)$ • The derivative function is used to find the slope of the tangent line of a function at any point • The derivative at a point tells us the rate of change at that exact point. This helps us solve problems in across all interdisciplinary fields • A family of functions can all have the same derivative (e.g. all functions of the form $y=2x + k$ where k is a constant have a derivative of 2) • An implicit function is one in which one variable cannot be expressed explicitly in terms of the other. • In many real-world scenarios, multiple rates are changing at the same time. For example, how fast is the volume of a melting snowball changing at the moment the radius is 20cm and decreasing at 3cm/min?
Areas of Focus: Proficiencies (Cumulative Progress Indicators)	**Examples, Outcomes, Assessments**
Students will: • Find the derivative of power, polynomial, exponential, logarithmic, and trigonometric functions	Instructional Focus: • 3.1 Powers and polynomials • 3.2 The exponential function

• Take the derivative of a constant multiples and sum and differences. • Apply the power, product, quotient, and chain rules • Apply implicit differentiation • Write the equation of tangent and normal lines • Determine the coordinates where functions have horizontal tangent lines • Interpret and solve real-world scenarios using derivative rules	• 3.3 The product and quotient rules • 3.4 The chain rule • 3.5 The trigonometric functions • 3.6 The chain rule and inverse functions • 3.7 Implicit functions • 4.6 Related rates Sample Assessments: • Classwork and homework • Exit tickets/warm-ups/short quizzes • Long quizzes • Unit exam Summative assessments/projects that encompass all skills learned in this unit can be found on page 184 in the text. These assessments can be given independently or as a group, and include 21st century themes and skills and higher-order thinking to complete. Instructional Strategies: **Interdisciplinary Connections** • Rate of change of acceleration and voltage problems (science) and inflation rates / value of assets (business/economics) **Technology Integration** • TI 83+, Desmos, Geogebra, Wolfram Alpha will assist in the calculation of derivatives, tangent and normal lines, related rates, and find the rate at which quantities are changing at specific points of interest given real-world data. **Global Perspectives/Culturally Responsive Teaching** • Using a variety of functions that model global temperatures, find the rate of change in temperature in specific years. Make predictions based on your findings

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Unit 4:
Applications of the Derivative

Concepts learned in Standard 2 and 3 are extended to incorporate a real-world application, optimization. The Mean Value Theorem and linearization are also examined. As a culminating activity, a weeklong economics-driven group project cements the concepts presented.

Big Ideas:

- Calculus is the study of change
- Derivatives help us understand events in the natural sciences and economics.

Essential Questions *What provocative questions will foster inquiry, understanding, and transfer of learning?*	**Enduring Understandings** *What will students understand about the big ideas?*
• How are average and instantaneous velocity connected? • How does differentiability make a graph look straight when it is zoomed in at a point? • How do derivatives help solve real-world problems?	• The Mean Value Theorem provides a way to show that if f is continuous on some interval, there is some number "c" such that the instantaneous rate of change at "c" is the same as the average rate of change on that interval (i.e. the slope of tangent and secant line are the same under these conditions) • Using the first and second derivative tests locate values of interest on functions that model real-world data. Local and global extrema and other behaviors are revealed from these tests. • L'Hopital's rule provides a way to deal with limits.
Areas of Focus: Proficiencies (Cumulative Progress Indicators)	**Examples, Outcomes, Assessments**
Students will: • Use the Mean Value Theorem and Rolle's Theorem to find values on specific intervals where the instantaneous rate of change is equal to the average rate of change. Create a function over an interval that does not satisfy the hypothesis of the MVT. • Use local linearization to find a tangent line approximation for functions near specific values	Instructional Focus: • 3.9 Linear approximation and the derivative • 3.10 Theorems about differentiable functions • 2.5/4.1 Using the first and second derivative • 4.2 Optimization • 4.3 Optimization and modeling • 4.7 L'Hopital's rule

• Use the first and second derivative tests to locate critical values and to answer questions about optimization • Find intervals of increase and decrease, and where a function is concave up or down. • Fuse geometric applications and formulas with calculus to solve problems involving related rates .	Sample Assessments: • Classwork and homework • Exit tickets/warm-ups/short quizzes • Long quizzes • Unit exam • Group Project/Presentation. In groups of 4-5, an object (cereal box, can of soup, etc.) will be analyzed to ensure the package is optimized. Then, two other containers will be examined to see if the current object is in the most cost-efficient container. This week-long project culminates in group presentations using Powerpoint or Prezi. Summative assessments/projects that encompass all skills learned in this unit can be found on pages 267-269 in the text. These assessments can be given independently or as a group, and include 21st century themes and skills and higher-order thinking to complete. Instructional Strategies: **Interdisciplinary Connections** • Applications include minimizing gas consumption, relationship between atmospheric pressure and altitude, and gravitational force. **Technology Integration** • TI 83+, Desmos, Geogebra, Wolfram Alpha will assist in the calculation of optimization problems.

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Unit 5:
Introduction to Integrals

Real-world examples are studied numerically, algebraically and graphically, to develop students' intuition behind what definite and indefinite integrals are how and they are used.

Big Ideas:

- Calculus is the study of change
- Integrals help us understand events in the natural sciences and economics.

Essential Questions *What provocative questions will foster inquiry, understanding, and transfer of learning?*	**Enduring Understandings** *What will students understand about the big ideas?*
• How do we measure distance traveled? • What is a Riemann sum and how does combining it with the concept of a limit help us solve problems? • What is a definite integral and how does it relate to other areas of calculus? • How are indefinite integrals related to derivatives?	• The area underneath a curve can be approximated by summing up the area of rectangles of varying bases and heights that go from the x-axis to the function itself. • Some estimates using rectangles are over estimates and some are under. This depends on whether or not the function is increasing or decreasing more prevalently and whether left or right end points are used • To get an exact area, the width of the rectangles used underneath the curve should go to zero (concept of a limit, unit 1) • The area underneath a velocity curve represents distance traveled • Area under the x-axis counts negatively and area above the x-axis counts positively • "Reversing" derivative rules allow you to find indefinite integrals
Areas of Focus: Proficiencies (Cumulative Progress Indicators)	**Examples, Outcomes, Assessments**
Students will: • Estimate distance traveled given tables and graphs by calculating left and right endpoint sums as well as trapezoid and midpoint calculations • Find the area under a curve of a familiar shape (semi-circles, rectangles, triangles, trapezoids, etc)	Instructional Focus: • 5.1 How do we measure distance traveled? • 5.2 The definite integral • 7.5 Numerical methods for definite integrals • 6.2 Constructing antiderivatives

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Use and interpret definite integral notation (upper and lower limits of integration, integrand)

Count areas above the x-axis as positive and areas below the x-axis as negative

Find indefinite integrals for constants, polynomials, “e”, and trigonometric functions

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Sample Assessments:

Classwork and homework
Exit tickets/warm-ups/short quizzes
Long quizzes
Unit exam

Summative assessments/projects that encompass all skills learned in this unit can be found on pages 316-318 and 350-352 in the text. These assessments can be given independently or as a group, and includes 21st century themes and skills and higher-order thinking to complete.

Instructional Strategies:
Interdisciplinary Connections

This introductory unit on integration is mostly skill-based but incorporates various acceleration problems

Technology Integration

TI 83+, Desmos, Geogebra, Wolfram Alpha will assist in the calculation of total distance traveled problems, in addition to preliminary problems involving calculating the exact value of definite integrals

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<td>The following skills and themes listed to the right should be reflected in the design of units and lessons for this course or content area.</td>
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21st Century Skills:
Creativity and Innovation

Critical Thinking and Problem Solving

Communication and Collaboration

Information Literacy

Media Literacy

Life and Career Skills

21st Century Themes (as applies to content area):
Financial, Economic, Business, and Entrepreneurial Literacy

Civic Literacy

Health Literacy
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	S.T.E.A.M. (Science, Technology, Engineering, Arts, Mathematics)

Unit 6:
Integration Techniques/Applications of Integration

The Fundamental Theorem of Calculus (both parts) and its applications are studied in depth. Methods of integration (including integration by substitution) are introduced both with and without a calculator. The unit culminates with applications of integration.

Big Ideas:

- Calculus is the study of change
- Integrals help us understand events in the natural sciences and economics.

Essential Questions *What provocative questions will foster inquiry, understanding, and transfer of learning?*	**Enduring Understandings** *What will students understand about the big ideas?*
• What is the importance of the Fundamental Theorem of Calculus (both parts)? • How does the concept of a differential equation tie into themes learned throughout calculus? • How can integration be used to solve geometry problems?	• The Fundamental Theorems of Calculus define the inverse relationship between the derivative and integral, as well as provide a method for computing a definite integral using antiderivatives. • The initial condition of a differential equation helps us pick the particular solution to the equation. • Washers, disks, and shells can be used to find the area of geometric figures, both two and three dimensional.

Areas of Focus: Proficiencies (Cumulative Progress Indicators)	**Examples, Outcomes, Assessments**
Students will:	Instructional Focus: <u>Integration Techniques</u> • 5.3 The Fundamental Theorem of Calculus (part 1) • 6.1 Constructing antiderivatives • 6.3 Differential Equations and motion • 6.4 The second Fundamental Theorem of Calculus • 7.1 Integration by substitution <u>Applications of Integration</u> • 8.1 Area and volume • 8.2 Applications to Geometry • Shell method (supplement)
• Use the FTC1 to evaluate definite integrals without a calculator	
• Use the FTC2 to evaluate definite integrals where the upper limit of integration is a function	
• Use differential equations and initial conditions to answer questions about position, velocity and acceleration of objects	
• Evaluate definite integrals where the integrand has an "inner" function (by substitution)	
• Find the area between curves and the x-axis and y-axis, and between two functions	

• Find the volume of solids formed by rotating areas around specific axes.	Sample Assessments: • Classwork and homework • Exit tickets/warm-ups/short quizzes • Long quizzes • Unit exam Summative assessments/projects that encompass all skills learned in this unit can be found on pages 316-318 and 350-352 in the text. These assessments can be given independently or as a group, and includes 21st century themes and skills and higher-order thinking to complete. Instructional Strategies: **Interdisciplinary Connections** • The findings of Aristotle, Newton, Descartes and Galileo provide motivation for the principle of inertia seen in the Differential Equation chapter. **Technology Integration** • TI 83+, Desmos, Geogebra, Wolfram Alpha will assist in the calculation definite integrals, differential equation problems, and harder volume problems.
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	Civic Literacy Health Literacy S.T.E.A.M. (Science, Technology, Engineering, Arts, Mathematics)

Texts and Resources:

Hughes-Hallett, Deborah et al. Calculus. 6th ed. Wiley, 2013.