

Summit Public Schools

Summit, New Jersey

Grade Level 8/ Content Area: Mathematics

Length of Course: Full Academic Year

Curriculum: Foundations of Algebra

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Anticipated Timetable for Algebra Fundamentals Quarter 1

Unit 1

Topic	Time Frame
Calculate Measures of Central Tendency	1
Prepare and Interpret Graphs	1
Represent Narrative Problems as Expressions and Equations	1
Model Relationships in Tables as Equations	1
Evaluate Numerical and Algebraic Expressions (Order of Operations)	1
Quiz 1	1
Perform Mathematical Calculations Involving Signed Numbers	2
Perform Mathematical Calculations Involving Rational Numbers	1
Add and Subtract Matrices	1
Multiply Matrices	2
Quiz 2	1
Test Review	1
Test – Tools of Algebra	1
Moving Straight Ahead Investigations 1 & 2	5
Total	20 days

Unit 2

Topic	Time Frame
Analyzing Data Using Scatter Plots	2
Relating Graphs to Events	2
Linking Graphs to Tables	1
Quiz 1	1
Functions	5
Writing a Function Rule	3
Three Views of a Function	1
Technology – Graphing Functions	1
Families of Functions	3
Quiz 2	1
Probability – Theoretical, experimental, with and without replacement	2
• Trend lines	1
• Statistics	1
Test Review	2
Test	1
Total	27 days

Quarter 2

Unit 3

Topic	Time Frame
Modeling and Solving One-Step Equations	1
Modeling and Solving Two-Step Equations	1
Modeling and Solving Equations with Algebra Tiles	1
Combining Like Terms To Solve Equations	1

Quiz 1	1
Distributive Property in Equations	2
Rational Numbers and Equations	2
Percent Equations	1
Percent of Change	1
Quiz 2	1
SIWS Investigation 1 (1.1,1.2,1.4)	3
SIWS Investigation 2 (2.1, 2.2)	2
SIWS Investigation 3 (3.1, 3.2)	2
Review of Unit	2
Unit Test	1
Total	22

Unit 4

Topic	Time Frame
Using Proportions	2
Equations with Variables on Both Sides	2
Using Graphs to Solve or Check Equations (graphing calculator)	1
Solving Absolute Value Equations	2
Transforming Formulas (Literal Equations)	2
Functions	1
Quiz Review	1
Quiz 4.1-4.4	1
Solving Inequalities Using Addition and Subtraction	2
Solving Inequalities Using Multiplication and Division	1
Solving Multi-Step Inequalities	2
Compound Inequalities	2
Using a Venn Diagram	1
Test Review	2
Test	1
Total	23 days

Quarter 3

Unit 5a

Topic	Time Frame
Determining Slope from a Graph	1
Calculating Slope Using the Slope Formula	1
Examining the Geometry of Slope	1
Determining Rates of Change from a Table	1
Identifying the Slope-Intercept Form of an Equation from a Graph	1
Graph Given the Slope-Intercept Form of an Equation	1
Quiz 1	1
Write an Equation of a Line Given the Slope and a Point	1
Write an Equation of a Line Given the Two Points	1
Identify the Line of Best Fit for a Scatter Plot	1
Write an Equation of a Line of Best Fit for a Scatter Plot	1
Graph Equations in General Linear Form	1

Write Equations for Parallel and Perpendicular Lines	1
Use Slope to Determine Whether Lines are Parallel or Perpendicular	1
Quiz 2	1
Test Review	1
Test – Graphs of Linear Equations	1
Total	17 days

Unit 5b

Topic	Time Frame
Use Formulas to Determine the Surface Areas of 3-Dimensional Shapes	2
Use Formulas to Determine Volume of 3-Dimensional Shapes	2
Determine the Missing Lengths of Similar Polygons	1
Determine the Missing Lengths of Congruent Polygons	1
Quiz 1	1
Use Dimensional Analysis to Convert from One Unit of Measure to Another	2
Thinking With Mathematical Models Investigation 1	2
Thinking With Mathematical Models Investigation 2	3
Quiz 2	1
Total	15 days

Quarter 4

Unit 6

Topic	Time Frame
Solving Systems by Graphing	2
Solving Systems using Substitution	2-3
Quiz 1	1
Solving Systems using Elimination	2
Writing Systems	2
Linear Inequalities	1
Systems of Linear Inequalities	2
Quiz 2	1
Linear Programming	2
Systems with Non-Linear Equations	2
Review Unit	2
Unit Test	1
Total	21

Discrete math unit

Topic	Time Frame
Combinations	3 days
Permutations	3 days
Counting Principal	1 day
Networks	2 days
Review Unit	1 day
Test	1 day
Total	11 days

Course Description: This course represents the first part of a two-year course. It has been designed to offer a rigorous and comprehensive foundation that addresses the newly expanded core content standards for Algebra 1. It will also provide students the opportunity to truly master algebraic and mathematics skills that will lead to greater achievement in subsequent courses. Students will create and use numerical, algebraic, graphical, and verbal representations and analyze sophisticated patterns, relations, and functions. They will represent linear functions numerically, algebraically, graphically and verbally and work with and interpret these representations. They will deepen their understanding of relations and functions and expand their repertoire in working with them. Students will develop insight and understanding of the algebraic properties that govern the manipulation of symbols in expressions, equations, and inequalities. Functions will be introduced as mathematical modeling tools providing students with a versatile and powerful means for analyzing and solving complex, multi-step, real world problems. Students will also learn the appropriate use of technology, such as graphing calculators and spreadsheet utilities to model and analyze a wide range of mathematical relationships.

Unit 1: Tools of Algebra

Standard O (Operations on Numbers and Expressions)	
Students will be able to perform operations with real numbers, algebraic expressions and matrices. Reasoning skills will be emphasized, including justification of results.	
Big Ideas: <i>Course Objectives / Content Statement(s)</i>	
<ul style="list-style-type: none"> • Reason quantitatively and perform increasingly complex calculations using the Order of Operations. • Add, subtract and multiply matrices by a scalar value. 	
Essential Questions	Enduring Understandings
<i>What provocative questions will foster inquiry, understanding, and transfer of learning?</i>	<i>What will students understand about the big ideas?</i>
<ul style="list-style-type: none"> • How does one apply the order of operations in complex, multi-step problems? • What are matrices and how are mathematical operations applied to them? 	Students will understand that... <ul style="list-style-type: none"> • The Order of Operations can be applied to more complex problems. • Matrices are an arrangement of data into sets of rows and columns. • Matrices can be combined and/or multiplied by a scalar value.
Areas of Focus: Proficiencies (Cumulative Progress Indicators)	Examples, Outcomes, Assessments
Students will:	Instructional Focus (3 weeks): <ul style="list-style-type: none"> • Use numbers and their properties from each of the following sets: Natural, Whole, Integer, Rational and Real. • Introduce two-dimensional mathematics using matrices. Sample Assessments:
O1.a Use properties of number systems within the set of real numbers to justify reasoning.	
O1.B2 Use matrices to represent and solve problems.	

	<ul style="list-style-type: none"> • Sample SCR Item: <i>Express -3.25 as an Integer over a Natural number</i> • Sample ECR Item: <i>Jim determined that $3*(12 - 4*2) = 48$. Evaluate Jim's answer using the Order of Operations.</i> • 1 unit quiz • 1 unit test <p>Instructional Strategies:</p> <ul style="list-style-type: none"> • Use Venn diagrams as a graphic organizer when defining number sets. • Use mnemonic devices to memorize the Order of Operations. <p>Technology Integration</p> <ul style="list-style-type: none"> • Use a scientific calculator to explore multi-step problems. Introduce advanced functions (grouping symbols and memory functions).
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Standard L (Linear Relationships)	
Students will be able to use words, tables, graphs and symbols to represent, analyze and model with linear functions.	
Big Ideas: <i>Course Objectives / Content Statement(s)</i>	
<ul style="list-style-type: none"> • Recognize, describe and represent linear relationships using words, tables, numerical patterns and equations. Translate among these representations. 	
Essential Questions	Enduring Understandings
<i>What provocative questions will foster inquiry, understanding, and transfer of learning?</i>	<i>What will students understand about the big ideas?</i>
<ul style="list-style-type: none"> • How do you determine the equation of a linear relationship given a table of values? 	<p>Students will understand that...</p> <ul style="list-style-type: none"> • Symbolic equations can be derived directly from a table representing an associated set of ordered pairs.
Areas of Focus: Proficiencies (Cumulative Progress Indicators)	Examples, Outcomes, Assessments
Students will:	Instructional Focus(3 days):
L1.a Recognize, describe and represent linear relationships using tables and equations.	<ul style="list-style-type: none"> • Identify and distinguish between the linear characteristics represented in tables. • Develop function rules as a means to test conjectures regarding patterns observed in tables of ordered pairs.
	Sample Assessments:

	<ul style="list-style-type: none"> • Complete Investigations 1 & 2 from CMP2 <u>Moving Straight Ahead</u> • 1 unit quiz <p>Interdisciplinary Connections</p> <ul style="list-style-type: none"> • Discuss the connection between Algebra and Science. Encourage students to identify applications of function rules in other content areas. <p>Global Perspectives</p> <ul style="list-style-type: none"> • Reflect with students the “universal” nature of mathematics and that mathematical patterns can be found throughout the world.
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Standard 4.4 (Data Analysis)	
Students will develop an understanding of the concepts and techniques of data analysis.	
Big Ideas: <i>Course Objectives / Content Statement(s)</i>	
<ul style="list-style-type: none"> • Understand and interpret data and data models representing a variety of real-world situations. 	
Essential Questions	Enduring Understandings
<i>What provocative questions will foster inquiry, understanding, and transfer of learning?</i>	<i>What will students understand about the big ideas?</i>
<ul style="list-style-type: none"> • Why does it make sense to examine all three measures of central tendency when analyzing a data set? 	Students will understand that... <ul style="list-style-type: none"> • Data extremes can distort sets of data.
Areas of Focus: Proficiencies (Cumulative Progress Indicators)	Examples, Outcomes, Assessments
Students will:	Instructional Focus (2 days):
4.4.8 A.1 Select and use appropriate representations for sets of data and measures of central tendency.	<ul style="list-style-type: none"> • Identify and distinguish the characteristics of mean, median, mode and range. • Explore statistical anomalies including outliers. <p>Sample Assessment:</p> <ul style="list-style-type: none"> • Sample ECR Item: <i>Which measure of central tendency (mean, median or mode) is a better measure for a given set of data? Explain your rationale.</i>

Unit 2: Functions and Their Graphs

Standard L (Linear Relationships)	
Students will be able to solve and graph the solution sets of linear equations, inequalities and systems of linear equations and to use words, tables, graphs and symbols to represent, analyze and model with linear functions.	
Big Ideas: <i>Course Objectives / Content Statement(s)</i>	
<ul style="list-style-type: none"> Recognize, describe and represent linear relationships using words, tables, numerical patterns, graphs and equations. Translate among these representations. Describe, analyze and use key characteristics of linear functions and their graphs. Graph the absolute value of a linear function and determine and analyze its key characteristics. Recognize, express and solve problems that can be modeled using linear functions. Interpret their solutions in terms of the context of the problem. 	
Essential Questions	Enduring Understandings
<i>What provocative questions will foster inquiry, understanding, and transfer of learning?</i>	<i>What will students understand about the big ideas?</i>
<ul style="list-style-type: none"> How can change be best represented mathematically? How can we use mathematical language to describe change? How can we use mathematical models to describe change or change over time? How can patterns, relations, and functions be used as tools to best describe and help explain real-life situations? How are patterns of change related to the behavior of functions? How are functions and their graphs related? How can technology be used to investigate properties of linear functions and their graphs? 	Students will understand that... <ul style="list-style-type: none"> Graphs and equations are alternative ways for depicting and analyzing patterns of change. Functional relationships can be expressed in real context, graphs, algebraic equations, tables, and words; each representation of a given function is simply a different way of expressing the same idea. The value of a particular representation depends on its purpose. A variety of families of functions can be used to model and solve real world situations.
Areas of Focus: Proficiencies (Cumulative Progress Indicators)	Examples, Outcomes, Assessments
Students will:	Instructional Focus:
L1.b Describe, analyze and use key characteristics of linear functions and their graphs.	<ul style="list-style-type: none"> Identifying and distinguishing among parameters and the independent and dependent variables in a linear relationship.
	Sample Assessments:
	<ul style="list-style-type: none"> 2 unit quizzes 1 unit test
	SCR: Write an equation for a line parallel to the line through (1, -2) and (-3, 5).

ECR: Compare and contrast the positions of the graphs for the following three functions and explain how the positions are related to the equations:

$$f(x) = 5x \quad g(x) = 5x + 2 \quad \text{and} \quad h(x) = 5x - 2$$

Performance Assessment Task: The Math Club needs to raise money for its annual neighborhood part beautification project. The club members decide to have a one-day car wash to raise money for this project. After estimating the cost of the activities, determine the total cost of sponges, rags, soap, buckets, and other materials, and investigate the average local charge for washing one car. Write a general rule to determine how much money can be raised for any number of cars. Realistically, can the car wash raise enough money to support this activity?

Instructional Strategies:

Interdisciplinary Connections

- Investigate the relationship between stopping distance and speed of travel in a car. Gather data from the driver's education manual or online through the Motor Vehicle Commission.

Technology Integration

- Graph the values found in the activity above, note that the relationship is linear, and look for an equation that fits the data.
- Use a graphing calculator to determine how changes to the equation affect its graph. Discuss the similarities and differences between different equations and graphs. Determine which elements of an equation cause changes in the graphs.

Standard D (Data, Statistics, and Probability)

Students will be able to apply algebraic knowledge to the interpretation and analysis of data,

<p>statistics and probability. Analysis and interpretation of univariate and bivariate data includes the use of summary statistics for sets of data and estimation of lines of best fit.</p>	
<p>Big Ideas: <i>Course Objectives / Content Statement(s)</i></p> <ul style="list-style-type: none"> • Interpret and compare linear models for data that exhibit a linear trend including contextual problems. 	
<p>Essential Questions</p> <p><i>What provocative questions will foster inquiry, understanding, and transfer of learning?</i></p>	<p>Enduring Understandings</p> <p><i>What will students understand about the big ideas?</i></p>
<ul style="list-style-type: none"> • How can the collection, organization, interpretation, and display of data be used to answer questions? • How can the representation of data influence decisions? • How can experimental and theoretical probabilities be used to make predictions or draw conclusions? 	<p>Students will understand that...</p> <ul style="list-style-type: none"> • The message conveyed by the data depends on the display. • Tables, charts, tree diagrams, and multiplication can be used to determine how many ways an event can occur. • Probability is about predictions over the long term rather than predictions of individual events.
<p>Areas of Focus: Proficiencies (Cumulative Progress Indicators)</p>	<p>Examples, Outcomes, Assessments</p>
<p>Students will:</p> <p>D1.a. Interpret and compare linear models for data that exhibit a linear trend in the context of a problem.</p>	<p>Instructional Focus:</p> <ul style="list-style-type: none"> • Creating scatter plots and estimating a line of best fit. • Using lines of best fit to extrapolate or interpolate within the range of the data and within the context of the problem. • Determine when, within the context of a problem, it may be unreasonable to extrapolate beyond a certain point. <p>Sample Assessments:</p> <ul style="list-style-type: none"> • 2 unit quizzes • 1 unit test <p>ECR: Given a set of points (1, 1), (2, 3), (4, 7), (6, 9), and (7, 13), if a sixth point were included in the set, which of the following would have the greatest impact, as the sixth point in the set, on the slope of a line of best fit? Justify your answer. (3, 4), (6, 10), (8, 23), or (10, 19)</p> <p>ECR: If a linear trend describes population growth in a small town over 5 years, explain why it would not be best to use the same linear trend to predict population in the town after 100 years.</p>

Unit 3 Algebraic Concepts and Simple Equations

Standard O: Operations on Numbers and Expressions

Successful students will be able to perform operations with algebraic expression.. Reasoning skills will be emphasized, including justification of results.

Big Ideas:

- **Students will use the distributive property to simplify expressions**
- **Students will identify and combine like terms to simplify expressions**

Essential Questions

What provocative questions will foster inquiry, understanding, and transfer of learning?

Enduring Understandings

What will students understand about the big ideas?

How can numeric operations be extended to algebraic objects?

Why is it useful to represent real-life situations algebraically?

What makes an algebraic algorithm both effective and efficient?

Students will understand that...

- Rules of arithmetic and algebra can be used together with (the concept of) equivalence to transform equations so solutions can be found to solve problems.
- Variables are symbols that take the place of numbers or ranges of numbers; they have different meanings depending on how they are being used.

Areas of Focus: Proficiencies (Cumulative Progress Indicators)

Examples, Outcomes, Assessments

Students will:

O1.B1 Describe and distinguish among the various uses of variables,

O2.b Add, subtract and multiply polynomial expressions.

Instructional Focus:

- Modeling and Solving One and Two step equations
- Combining Like Terms to Solve Equations
- Distributive Property in Equations
- Percent Equations
- Percent Change
- Writing Equivalent Expressions and Equations

Sample Assessments: Unit Quiz / Test

- Solve and Check the following equation: $6(y + 3) = 24$
- Find the % mark-up if a stores sells a \$23 shirt for \$30.
- Every Saturday you play basketball in the local community youth club. At the end of a season after a club

tournament, the players in the club meet at a fast-food restaurant for a party. If hamburgers cost 59 cents each, find a way to determine the total cost of hamburgers when various numbers of players in the club each have a hamburger (NCTM *Illuminations*).

Instructional Strategies:

Interdisciplinary Connections

Use % Change to find and compare the rate of inflation throughout the 20th century for the United States and Great Britain. Results can be presented to the class.

Make a model of the relationship between Celsius and Fahrenheit temperatures. Represent the relationship as an equation, and check the equation against two known data points – 0 degrees C = 32 degrees F and 100 degrees C = 212 degrees F. Use the equation to convert between Celsius and Fahrenheit temperatures.

Technology Integration

Using spreadsheet software, examine variables as a set of objects and find the image of a set of objects using a function to gain an output. For example, the corresponding values of $f(n) = 3n$ are examined by using a single number substituted for n in the function, next by using the set of natural numbers less than or equal to 50 under this function, and finally to considering the variable n as the set of real numbers. Students identify a real world situation where a continuous function using the set of real numbers versus a single number may occur (e.g. the gravitational force on an object of a particular mass as it moves to higher altitudes which might include either mountain climbing or a space shuttle trip; the height of a candle as it burns over

	<p>time).</p> <p>Global Perspectives Compare and contrast populations of various nations to determine % growth or decay. Students will determine other environmental or political factors that affect the population changes.</p>
<p>The following skills and themes listed to the right should be reflected in the design of units and lessons for this course or content area.</p>	<p>21st Century Skills:</p> <ul style="list-style-type: none"> Creativity and Innovation Critical Thinking and Problem Solving Communication and Collaboration Information Literacy Media Literacy Life and Career Skills <p>21st Century Themes (as applies to content area):</p> <ul style="list-style-type: none"> Financial, Economic, Business, and Entrepreneurial Literacy Civic Literacy Health Literacy
<p>Standard L: Linear Relationships</p> <p>Successful students will be able to solve and graph the solution sets of linear equations, and to use words, tables, graphs, and symbols to represent, analyze, and model with linear functions. Function notation should be introduced and used regularly but not exclusively.</p>	
<p>Big Ideas:</p> <ul style="list-style-type: none"> • Students will solve one and two step equations • Students will use the distributive property and like terms to simplify and solve equations and expressions 	
<p>Essential Questions</p> <p><i>What provocative questions will foster inquiry, understanding, and transfer of learning?</i></p>	<p>Enduring Understandings</p> <p><i>What will students understand about the big ideas?</i></p>
<p>How can patterns, relations, and functions be used as tools to best describe and help explain real-life situations?</p>	<p>Students will understand that...</p> <ul style="list-style-type: none"> • Graphs and equations are alternative (and often equivalent) ways for depicting and analyzing patterns of change. • The value of a particular

	representation depends on its purpose.
Areas of Focus: Proficiencies (Cumulative Progress Indicators)	Examples, Outcomes, Assessments
Students will:	Instructional Focus:
L2.a Solve single-variable linear equations and inequalities with rational coefficients.	<ul style="list-style-type: none"> • Describing and distinguishing among the types of equations that can be constructed by equating linear expressions: <ul style="list-style-type: none"> ○ Identities (such as $x + 0 = x$) ○ Equations for which there is no solution (such as $x + 3 = x$) ○ Formulas ○ Equations where the solution is unique • Solving multi-step equations and inequalities. • Interpreting solutions in terms of the context of the problem. • Using and interpreting appropriate units of measurement, estimation and the appropriate level of precision for applications.
L2.e Recognize, express and solve problems that can be modeled using single-variable linear equations; one- or two-variable inequalities; or two-variable systems of linear equations.	<p>Sample Assessments:</p> <p>MC: Which of the following equations has no solution?</p> <p>A. $x + 0 = x$ B. $y + 1 = 2x$ * C. $x + 4 = x$ D. $3x = 9$</p> <p>MC (Calculator Permitted):</p> <p>Sandra’s property has the shape of a trapezoid with the dimensions shown. If the perimeter of the property is 3,279 feet, what is the value of x?</p> <p>A. 726 ft <i>Solution:</i> $375 + 4x = 3279$ B. 781.25 ft $x = 726$ C. 913.5 ft D. 1452 ft</p> <p>SCR (Calculator Permitted): Jim spent \$200 on gifts for his family. He spent the</p>

money on toys, clothes, and a \$15 DVD. He spent 4 times as much on clothes as he did on toys. Write an equation in one variable that can be used to determine how much money Jim spent on toys. Solve the equation to determine how much Jim spent on toys.

(Sample Solution:

$$\text{Let } t = \text{money spent on toys} \rightarrow 4t + t + 15 = 200$$

$$5t = 185$$

$$t = 37)$$

SCR (Calculator Permitted): The measure of one angle of an acute triangle is twice the measure of the first angle while the third is 30° more than the first angle. Determine the measures of the three angles.

(Sample Solution:

Instructional Strategies:

Interdisciplinary Connections

Make a model of the relationship between Celsius and Fahrenheit temperatures. Represent the relationship as an equation, and check the equation against two known data points – 0 degrees C = 32 degrees F and 100 degrees C = 212 degrees F. Use the equation to convert between Celsius and Fahrenheit temperatures.

Use Hooke's Law, $F = dk$, to determine the constant k when force is being applied to a spring and the distance of the stretch of the spring is known.

Technology Integration

Using spreadsheet software, examine variables as a set of objects and find the image of a set of objects using a function to gain an output. For example, the corresponding values of $f(n) = 3n$ are examined by using a single number

	<p>substituted for n in the function, next by using the set of natural numbers less than or equal to 50 under this function, and finally to considering the variable n as the set of real numbers. Students identify a real world situation where a continuous function using the set of real numbers versus a single number may occur (e.g. the gravitational force on an object of a particular mass as it moves to higher altitudes which might include either mountain climbing or a space shuttle trip; the height of a candle as it burns over time).</p> <p>Global Perspectives</p> <p>Compare and contrast populations of various nations to determine % growth or decay. Students will determine other environmental or political factors that affect the population changes.</p>
<p>The following skills and themes listed to the right should be reflected in the design of units and lessons for this course or content area.</p>	<p>21st Century Skills:</p> <ul style="list-style-type: none"> Creativity and Innovation Critical Thinking and Problem Solving Communication and Collaboration Information Literacy Media Literacy Life and Career Skills <p>21st Century Themes (as applies to content area):</p> <ul style="list-style-type: none"> Financial, Economic, Business, and Entrepreneurial Literacy Civic Literacy Health Literacy

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Unit 4: Equations and Inequalities

<p style="text-align: center;">Standard O (Operations on Numbers and Expressions)</p> <p>Students will be able to perform operations with real numbers and algebraic expressions, including expressions involving exponents, scientific notation, and square roots, using estimation and an appropriate level of precision. Reasoning skills will be emphasized, including justification of results.</p>	
<p>Big Ideas: <i>Course Objectives / Content Statement(s)</i></p> <ul style="list-style-type: none"> Use rates, ratios and proportions to solve problems, including measurement problems. 	
<p style="text-align: center;">Essential Questions</p> <p style="text-align: center;"><i>What provocative questions will foster inquiry, understanding, and transfer of learning?</i></p>	<p style="text-align: center;">Enduring Understandings</p> <p style="text-align: center;"><i>What will students understand about the big ideas?</i></p>
<ul style="list-style-type: none"> How can numeric operations be extended to algebraic objects? 	<p>Students will understand that...</p> <ul style="list-style-type: none"> Proportionality involves a relationship in which the ratio of two quantities remains constant as the corresponding values of the quantities change.
<p style="text-align: center;">Areas of Focus: Proficiencies (Cumulative Progress Indicators)</p>	<p style="text-align: center;">Examples, Outcomes, Assessments</p>
<p>Students will:</p> <p>O1.b Use rates, ratios and proportions to solve problems, including measurement problems.</p>	<p>Instructional Focus:</p> <ul style="list-style-type: none"> Solving problems using derived measures (e.g. percent change and density) Solving problems involving scale factor (e.g. similar figures, scale drawings, map scales). Solving applications related to proportional representation. <p>Sample Assessments:</p> <ul style="list-style-type: none"> 2 unit quizzes 1 unit test

Performance Assessment Task: Suppose that a drug company has established that a patient must have 40 mg of a certain prescription drug in the body for the drug to be effective. Moreover, the company's studies indicate that anything in excess of 600 mg is toxic, and its research has shown that the body eliminates 10 percent of the drug every four hours. Imagine you are a doctor prescribing this drug for a patient. How often would you want your patient to take the drug, and in what quantity, to ensure effectiveness while avoiding toxicity?

Instructional Strategies:

Core Mathematical Process – Problem Solving : There are 248 students in the freshman class, 199 in the sophomore class, 158 in the junior class, and 97 in the senior class. The student council has 30 members, with these seats allocated based on the number of students in each class. How many student council members should each class have?

Interdisciplinary Connections: Find a linear relationship between the length of a person's thigh bone and his height, and use this to estimate the height of a person whose thigh bone has been found in an archeological dig.

Interdisciplinary Connections: Have students consider the advantages and disadvantages of different voting methods, including weighted voting (ranking 1st, 2nd, and 3rd choices), runoff elections, plurality voting (whoever gets the most votes wins) and majority voting (winner must receive more than half of the votes). Link this discussions to social studies and the study of government.

Standard L (Linear Relationships)

Students will be able to solve and graph the solution sets of linear equations, inequalities and systems of linear equations and to use words, tables, graphs and symbols to represent, analyze and model with linear functions.

Big Ideas: *Course Objectives / Content Statement(s)*

- Solve single variable linear equations and inequalities with rational coefficients.

<ul style="list-style-type: none"> • Solve equations involving the absolute value of a linear expression. • Recognize, express and solve problems that can be modeled using single-variable linear equations and one- or two-variable inequalities. Interpret their solutions in terms of the context of the problem. 	
Essential Questions <i>What provocative questions will foster inquiry, understanding, and transfer of learning?</i>	Enduring Understandings <i>What will students understand about the big ideas?</i>
<ul style="list-style-type: none"> • How can change be best represented mathematically? • How can we use mathematical language to describe change? • How can we use mathematical models to describe change or change over time? • How can patterns, relations, and functions be used as tools to best describe and help explain real-life situations? • How are patterns of change related to the behavior of functions? • How are functions and their graphs related? • How can technology be used to investigate properties of linear functions and their graphs? 	<p>Students will understand that...</p> <ul style="list-style-type: none"> • Graphs and equations are alternative ways for depicting and analyzing patterns of change. • Functional relationships can be expressed in real context, graphs, algebraic equations, tables, and words; each representation of a given function is simply a different way of expressing the same idea. • The value of a particular representation depends on its purpose. • A variety of families of functions can be used to model and solve real world situations.
Areas of Focus: Proficiencies (Cumulative Progress Indicators)	Examples, Outcomes, Assessments
<p>Students will:</p> <p>L2.a Solve single variable linear equations and inequalities with rational coefficients.</p> <p>L2.b Solve equations involving the absolute value of a linear expression.</p>	<p>Instructional Focus:</p> <ul style="list-style-type: none"> • Describing and distinguishing among the types of equations that can be constructed by equating linear expressions. <ul style="list-style-type: none"> ○ Identities ○ Equations for which there is no solution ○ Formulas ○ Equations where the solution is unique ○ Equations relating two variables. • Solving multi-step equations and inequalities. • Representing solution sets for inequalities symbolically as intervals or graphically on a number line. • Determine all possible values in the solution of an absolute value equation or inequality.

	<p>Sample Assessments:</p> <ul style="list-style-type: none"> • 2 unit quizzes • 1 unit test <p>MC: Which of the following equations has no solution?</p> <ol style="list-style-type: none"> $x + 0 = x$ $y + 1 = 2x$ $x + 4 = x$ $3x = 9$ <p>SCR: Solve $3 - x < 5$</p> <p>ECR: Determine and explain the solutions for each of the following three equations:</p> <ol style="list-style-type: none"> $x + 0 = x + 2$ $x + 0 = x$ $x + 0 = 2x$ <p>Technology Integration</p> <p>Global Perspectives</p> <ul style="list-style-type: none"> • The probabilities of a child being a boy or girl are not exactly the same. Examine what factors, worldwide, explain that, though there are 106 boys born for every 100 girls, girls soon outnumber boys.
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<p>Standard N2 (Non-linear Relationships)</p> <p>Students will be able to recognize, represent, analyze, graph, solve equations and apply some non-linear functions, including quadratic and exponential. There are a variety of types of test items including some that cut across the objectives in this standard and require students to make connections and, where appropriate, solve contextual problems. In contextual problems students will be required to graph and interpret their solutions in terms of the context. They should be able to apply such problem solving heuristics as: identifying missing or irrelevant information, testing ideas; considering analogous or special cases; making appropriate estimates; using inductive or deductive reasoning; analyzing situations using symbols, tables, graphs, or diagrams; evaluating progress regularly; checking for reasonableness of results; using technology appropriately; deriving independent methods to verify results; and using the symbols and terms of mathematics correctly and precisely. Function notation should be introduced and used regularly but not exclusively.</p>
<p>Big Ideas: <i>Course Objectives / Content Statement(s)</i></p>

<ul style="list-style-type: none"> Solve equations involving several variables for one variable in terms of the others. 	
Essential Questions <i>What provocative questions will foster inquiry, understanding, and transfer of learning?</i>	Enduring Understandings <i>What will students understand about the big ideas?</i>
<ul style="list-style-type: none"> How can we use mathematical language to describe non-linear change? How can we model situations using quadratics? How can we model a situation using exponents? 	Students will understand that... <ul style="list-style-type: none"> Graphs and equations are alternative (and often equivalent) ways for depicting and analyzing patterns of non-linear change. Mathematical models can be used to describe physical relationships; these relationships are often non-linear. Real world situations, involving quadratic or exponential relationships, can be solved using multiple representations.
Areas of Focus: Proficiencies (Cumulative Progress Indicators)	Examples, Outcomes, Assessments
Students will: N2.a Solve equations involving several variables for one variable in terms of the others.	Instructional Focus: <ul style="list-style-type: none"> Understanding that solving a literal equation follows the same rules as solving any other equation. Sample Assessments: SCR: Solve for r : $V = \pi r^2 h$. SCR: Solve for y : $z = 3x^2y + 4y$

Unit 5: Graphing and Writing Linear Equations Surface Area & Volume of Three-Dimensional Objects

Standard L (Linear Relationships)	
Students will be able to solve and graph the solution sets of linear equations, inequalities and systems of linear equations and to use words, tables, graphs and symbols to represent, analyze and model with linear functions.	
Big Ideas: <i>Course Objectives / Content Statement(s)</i> <ul style="list-style-type: none"> Recognize, describe and represent linear relationships using words, tables, numerical patterns, graphs and equations. Translate among these representations. Describe, analyze and use key characteristics of linear functions and their graphs. Recognize, express and solve problems that can be modeled using linear functions. 	
Essential Questions <i>What provocative questions will foster inquiry, understanding, and transfer of learning?</i>	Enduring Understandings <i>What will students understand about the big ideas?</i>
<ul style="list-style-type: none"> How is the slope of linear relationships reflected in a table of 	Students will understand that... <ul style="list-style-type: none"> Slope is the ratio of vertical change

<p>values, in an equation and on a graph?</p> <ul style="list-style-type: none"> • What are the three symbolic forms of linear equations and which is the best selection given specific input? • How do you determine the equation of a linear relationship given a table of values or a graph? • How are slopes of perpendicular and parallel lines represented in symbolic equations? • How does graphing linear equations apply to real world problems? • How do scatter plots relate to linear models? 	<p>to horizontal change.</p> <ul style="list-style-type: none"> • The graphical representation of each linear equation is a reflection of two characteristics (slope and y-intercept). • Symbolic equations can be derived directly from a linear graph or through an associated set of ordered pairs. • Slopes of perpendicular and parallel lines have unique relationships. • Graphs of linear relationships represent patterns and facilitate predictions in real-world settings (including scatter-plots).
<p>Areas of Focus: Proficiencies (Cumulative Progress Indicators)</p>	<p>Examples, Outcomes, Assessments</p>
<p>Students will:</p> <p>L1.a Recognize, describe and represent linear relationships using tables, graphs and equations.</p> <p>L1.b Describe, analyze and use key characteristics of linear functions and their graphs.</p> <p>L1.d Recognize, express and solve problems that can be modeled using linear functions. Interpret solutions in terms of the context of the problem.</p>	<p>Instructional Focus (3 weeks):</p> <ul style="list-style-type: none"> • Identify and distinguish the characteristics of slope and y-intercept as represented in tables, equations and graphs. • Emphasize the significant applications of this unit in working with related disciplines and in solving real-world problems. <p>Sample Assessments:</p> <ul style="list-style-type: none"> • Sample Multiple Choice (MC) Item: <i>Match a given graph to one of four possible equations in slope-intercept form.</i> • Complete Investigations 1 & 2 from CMP2 <u>Thinking with Mathematical Models</u>. • 2 unit quizzes • 1 unit test <p>Interdisciplinary Connections</p> <ul style="list-style-type: none"> • Discuss the connection between Algebra and Physical Science. Encourage students to identify applications of graphing in two-dimensions in other content areas. <p>Technology Integration</p> <ul style="list-style-type: none"> • Use a graphing calculator (hand held, computer-based or online) to determine how changes to an

	<p>equation affect its graph. Discuss the similarities and differences between various equations and graphs. Determine which elements of an equation correspond to specific characteristics of the equation's graph.</p> <p>Global Perspectives</p> <ul style="list-style-type: none"> • Reflect with students the “universal” nature of mathematics and that symbolic and graphical representations of mathematical relationships transcend most language and cultural boundaries.
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Standard 4.2 (Geometry and Measurement)	
Students will develop spatial sense and the ability to use geometric properties, relationships, and measurement to model, describe and analyze a variety of mathematical relationships.	
<p>Big Ideas: <i>Course Objectives / Content Statement(s)</i></p> <ul style="list-style-type: none"> • Spatial sense and geometric relationships are a means to solve problems and gain insight to a variety of mathematical relationships. • Measurement is a tool to quantify physical mathematical relationships. 	
Essential Questions <i>What provocative questions will foster inquiry, understanding, and transfer of learning?</i>	Enduring Understandings <i>What will students understand about the big ideas?</i>
<ul style="list-style-type: none"> • How do the size and shape of similar and congruent polygons relate to one another? • How does a two-dimensional representation of a three-dimensional object facilitate the calculation of surface area? • How do the various formulas for Volume relate to each other? 	Students will understand that... <ul style="list-style-type: none"> • Corresponding sides of similar polygons form proportions. • Surface area is the sum of areas of the sides of the object and may best be represented as a two-dimensional model. • Most volume calculations relate to the relationship between Base-Area and Height.
Areas of Focus: Proficiencies (Cumulative Progress Indicators)	Examples, Outcomes, Assessments
Students will: <ul style="list-style-type: none"> 4.2.8 A.4 Describe and compare geometric shapes and make conjectures regarding symmetry, congruence, and similarity. 4.2.8 A.7 Create two-dimensional representations for the surfaces of three-dimensional objects. 4.2.8 E.2 Recognize that the volume of a 	Instructional Focus (6 days): <ul style="list-style-type: none"> • Identify and distinguish the characteristics of three-dimensional objects. • Emphasize the selection of appropriate formulas for use in surface area and volume calculations.

pyramid or cone is one-third the volume of the similar prism or cylinder.	<p>Sample Assessments:</p> <ul style="list-style-type: none"> • Sample ECR Item: <i>Determine the ratio of the volume of two spheres whereby the radius of the larger sphere is three times greater than the radius of the smaller sphere.</i> • 2 unit quizzes <p>Interdisciplinary Connections</p> <ul style="list-style-type: none"> • Explore the connection between two and three-dimensional models with respect to Science and Industrial Technology programs.
4.2.8 E.3 Develop and apply strategies and formulas for finding the surface area and volume of three-dimensional figures.	
4.2.8 E.4 Use formulas for finding the surface area and volume of a sphere.	

Standard O (Operations on Numbers and Expressions)	
Students will be able to perform operations with real numbers and algebraic expressions. Reasoning skills will be emphasized, including justification of results.	
Big Ideas: <i>Course Objectives / Content Statement(s)</i>	
<ul style="list-style-type: none"> • Reason quantitatively and use units to solve problems. 	
Essential Questions	Enduring Understandings
<i>What provocative questions will foster inquiry, understanding, and transfer of learning?</i>	<i>What will students understand about the big ideas?</i>
<ul style="list-style-type: none"> • How do unit labels facilitate the conversion of a measurement from one unit to another? 	<p>Students will understand that...</p> <ul style="list-style-type: none"> • Conversion calculations reflect properties related to the multiplication of rational expressions.
Areas of Focus: Proficiencies (Cumulative Progress Indicators)	Examples, Outcomes, Assessments
Students will:	Instructional Focus (2 days):
O1.b Use rates, ratios and proportions to solve problems, including measurement problems.	<ul style="list-style-type: none"> • Use dimensional analysis for unit conversion. <p>Sample Assessments:</p> <ul style="list-style-type: none"> • Sample SCR Item: <i>Convert 55 miles per hour to “n” feet per second.</i> • Integrated within other assessments. <p>Interdisciplinary Connections</p> <ul style="list-style-type: none"> • Explore the significant application that dimensional analysis has in Physics (unit conversion) and Chemistry (molecular calculations).

Unit 6 Systems of Equations and Inequalities

Standard L: Linear Relationships	
<p>Successful students will be able to solve and graph the solution sets of linear equations, inequalities and systems of linear equations and to use words, tables, graphs, and symbols to represent, analyze, and model with linear functions.</p>	
<p>Big Ideas:</p> <ul style="list-style-type: none"> • Students will solve systems of equations graphically and algebraically. • Students will solve linear inequalities graphically. • Students will solve systems of inequalities graphically. 	
Essential Questions	Enduring Understandings
<i>What provocative questions will foster inquiry, understanding, and transfer of learning?</i>	<i>What will students understand about the big ideas?</i>
<p>How are functions and their graphs related?</p> <p>How can technology be used to investigate properties of linear functions and their graphs?</p> <p>How can systems of equations be used to solve real-life situations?</p>	<p>Students will understand that...</p> <ul style="list-style-type: none"> • Graphs and equations are alternative (and often equivalent) ways for depicting and analyzing patterns of change. • Functional relationships can be expressed in real contexts, graphs, algebraic equations, tables, and words; each representation of a given function is simply a different way of expressing the same idea. • The value of a particular representation depends on its purpose. • A variety of families of functions can be used to model and solve real world situations.
Areas of Focus: Proficiencies (Cumulative Progress Indicators)	Examples, Outcomes, Assessments
Students will:	Instructional Focus:
L1.b Describe, analyze and use key characteristics of linear functions and their graphs	<ul style="list-style-type: none"> • Interpreting slopes of given lines to determine whether lines are parallel, perpendicular, intersecting, or coincident.
L2.c Graph and analyze the graph of the solution set of a two-variable linear inequality.	<ul style="list-style-type: none"> • Representing algebraic solutions graphically on the coordinate plane. • Using a shaded half-plane with solid or open boundary for graphs of two-variable inequalities.
L2.d Solve systems of linear equations in two variables using algebraic and graphic procedures.	<ul style="list-style-type: none"> • Providing examples of ordered pairs that are included in the solution set of a two-variable linear inequality.
L2.e Recognize, express and solve problems that can be modeled using single-variable linear equations; one- or two-	<ul style="list-style-type: none"> • Solving systems of linear equations using substitution, elimination, and graphing.

variable inequalities; or two-variable systems of linear equations.

- Interpreting solutions in terms of the context of the problem.
- Using and interpreting appropriate units of measurement, estimation and the appropriate level of precision for applications.

Sample Assessments: Unit Quiz / Test

- Write an equation for a line parallel to the line through (1, -2) and (-3, 5).
- Graph $5x - y > 3$
- Graph $2x - 4y \leq 1$
- Determine a point in the solution set for $3x + 2y < 6$ by graphing.
- Solve the linear system by the method that you think is best. Show or explain your work. Explain why you chose that method.

$$\begin{aligned}7x - 8y &= 6 \\4x + y &= 9\end{aligned}$$

- Jim spent \$200 on gifts for his family. He spent the money on toys, clothes, and a \$15 DVD. He spent 4 times as much on clothes as he did on toys. Write an equation in one variable that can be used to determine how much money Jim spent on toys. Solve the equation to determine how much Jim spent on toys.
- A triangle is formed by the intersections of the x-axis, the y-axis, and the line $2x + 3y = 6$. What is the area of the triangle?

A. $\frac{2}{3}$ B. 2 C. 3 D. 6
- The measure of one angle of an acute triangle is twice the measure of the first angle while the third is 30° more than the first angle.

	<p>Determine the measures of the three angles.</p> <p>Instructional Strategies:</p> <p>Interdisciplinary Connections</p> <p>Make a model of the relationship between Celsius and Fahrenheit temperatures. Represent the relationship as an equation, and check the equation against two known data points – 0 degrees C = 32 degrees F and 100 degrees C = 212 degrees F. Use the equation to convert between Celsius and Fahrenheit temperatures.</p> <p>Technology Integration</p> <p>The Cape May-Lewes Ferry has space for cars and buses. Using the internet, investigate how many of each can be transported on a single trip. Use variables to represent the unknowns (e.g. x for cars and y for buses) and develop the graph of the inequality, using either paper-and-pencil or a graphing calculator. Recognizing that the solutions have to be whole numbers, students should identify the points whose coefficients are non-negative integers and in the first quadrant on or below the line.</p> <p>.</p>
<p>The following skills and themes listed to the right should be reflected in the design of units and lessons for this course or content area.</p>	<p>21st Century Skills:</p> <ul style="list-style-type: none"> Creativity and Innovation Critical Thinking and Problem Solving Communication and Collaboration Information Literacy Media Literacy Life and Career Skills <p>21st Century Themes (as applies to content area):</p>

	<p>Financial, Economic, Business, and Entrepreneurial Literacy</p> <p>Civic Literacy</p> <p>Health Literacy</p>
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Discrete Math Unit

Standard D 2 (Probability)	
Students will be able to apply algebraic knowledge to the interpretation and analysis of data, statistics and probability. Analysis and interpretation of univariate and bivariate data includes the use of summary statistics for sets of data and estimation of lines of best fit.	
Big Ideas: <i>Course Objectives / Content Statement(s)</i>	
<ul style="list-style-type: none"> • Use counting principles to determine the number of ways an event can occur. Interpret and justify solutions. • Apply probability concepts to determine the likelihood an event will occur in practical situations. 	
Essential Questions <i>What provocative questions will foster inquiry, understanding, and transfer of learning?</i>	Enduring Understandings <i>What will students understand about the big ideas?</i>
<ul style="list-style-type: none"> • When does order matter? • How can experimental and theoretical probabilities be used to make predictions or draw conclusions? 	<p>Students will understand that...</p> <ul style="list-style-type: none"> • The results of a statistical investigation can be used to support or refute an argument. • Tables, charts, tree diagrams, and multiplication can be used to determine how many ways an event can occur. • Probability is about predictions over the long term rather than predictions of individual events.
Areas of Focus: Proficiencies (Cumulative Progress Indicators)	Examples, Outcomes, Assessments
Students will:	Instructional Focus:
D2.a. Use counting principles to determine the number of ways an event can occur. Interpret and justify solutions.	<ul style="list-style-type: none"> • Using an understanding of permutations and combinations to solve problems with and without replacement. <p>Sample Assessments:</p> <ul style="list-style-type: none"> • 1 unit quiz • 1 unit test <p>ECR: (Non-calculator) Compare the number of ways the letters of the words FROG and DEER can be arranged to form unique four-letter sequences. Explain your answer.</p>

	<p>SCR: If a person has twice as many shirts as pairs of pants, how many different combinations can be made of a shirt and pair of pants, based on the number of pants?</p>
<p>D2b. Apply probability concepts to determine the likelihood an event will occur in practical situations.</p>	<p>Instructional Focus:</p> <ul style="list-style-type: none"> • Determining exactly or approximately, the probability that an event will occur based on simple experiments (e.g. tossing number cubes, flipping coins, spinning spinners), counting principles, or data. • Making predictions based on experimental and theoretical probabilities and comparing results. <p>Sample Assessments:</p> <p>SCR: If there are 4 brown, 4 black, and 4 blue socks in a drawer, what is the probability that a matched pair will be selected when drawing out first one then another, without replacing the first sock or being able to see the socks as they are drawn?</p> <p>SCR: In a sample of 100 randomly selected students, 37 of them could identify the difference in two brands of soft drinks. Based on these data, what is the best estimate of how many of the 2352 students in the school could distinguish between the soft drinks?</p>
<p>D2.C1 Determine and apply probabilities in complex situations.</p>	<p>Instructional Focus:</p> <ul style="list-style-type: none"> • Employing multiple representations (Venn diagrams, trees, tables, and area models) to analyze and summarize information concerning compound events <ul style="list-style-type: none"> ○ Mutually exclusive or not ○ Complementary events ○ Dependent or independent ○ Multiplication rule • Determining conditional probability. • Solving problems involving probability with simulations (using spinners, dice, calculators, and computers) and theoretical models. • Recognizing that simulation results are likely to differ from one run of the simulation to the next; observe that the

results of the simulation tend to converge as the number of trials increases (Law of Large Numbers)

- Evaluating medical test results and treatment options, analyzing risk in situations where anecdotal evidence is provided, interpreting media reports and evaluate conclusions.

Sample Assessments:

ECR: Jim has tossed a coin 8 times and gotten heads every time. He thinks that he is more likely to get tails on the next flip. Do you agree or disagree? Explain.

ECR: Ann is considering two different investments. The first investment is a stock which has a 25% chance of returning 10%, a 25% chance of returning 4%, and a 50% chance of losing 2%. The second investment is guaranteed to earn 5%. Which investment should she make. Justify your response.

Instructional Strategies:

Employ Venn diagrams to summarize information concerning compound events.

Interdisciplinary Connections:

Use probability to interpret odds and risks of financial investment options and recognize common misconceptions. Students investigate return and risk for various investments, including certificates of deposit, stocks, bonds, and real estate.

Interdisciplinary Connections:

Analyze the risks associated with a particular accident, illness, or course of treatment expressed as a probability. Present various incidents that have a similar probability of occurrence.

Texts and Resources:

Algebra: Tools for a Changing World (Prentice-Hall)

Standards:

New Jersey Core Curriculum Content Standards for Mathematics (adopted 1/9/2008)

Proposed New Jersey Algebra I Core Content (rev. 4/14/2010)

<http://www.state.nj.us/education/aps/cccs/math/alg1content.pdf>